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Abstract

In this paper we simulate the effects of price shifts on the profitability of coal and gas fired power plants in the German electricity market. The value of a power plant will be determined by a call option on the clean spread. We use a structural approach to connect fundamental factors, such as demand, capacity, coal and gas prices and emission allowances to the price of electricity. Based on empirical bid behaviour of electricity producers we develop a novel CO_2 emission rate model and deduce a partial differential equation to price emission allowances, including a feedback of carbon prices on the CO_2 emission rate.

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1 Introduction

Over the course of the last two decades the liberalization of energy markets has accelerated. While in the past energy companies could act independently of competition with fixed prices, they now have to face variable prices. With the foundation of the Federal Network Agency, which guarantees grid access under fair conditions, the variety and quantity of market participants increased. This development led to more intense competition, also affecting fuel markets and the volatility of input costs. Since the installation of the *European Union Emission Trading System* (EU ETS) another impact factor has been introduced. As a power plant manager has to surrender enough *European Emission Allowances* (EUAs) to cover the emitted greenhouse gas, they can be interpreted as part of production costs, thus influencing the margin of a power plant. We see, that in opposition to the past, investment decisions concerning new technologies or power plants now have to consider a more complex market situation and uncertainty about future price developments.

In this paper we want to quantify the value of coal and gas fired power plants in the German electricity market. Beside renewable energy they form a major part of the German energy mix. The profitability of a certain plant will be determined by the spread between the price of one unit of electricity and its production costs. If we neglect operational constraints, such as starting and minimum operating times, a plant will be online whenever its spread is positive. Therefore, the value can be computed with the help of a real option approach, the so-called spread options. Carmona et al. (2012) introduced a joint model to connect the price for electricity, fuel and emission allowances for a theoretical market. Inspired by their approach, we introduce a joint model for the complicated German energy market. Based on the stochastic bid stack, presented by Howison and Coulon (2009), we calculate the price for electricity and the current emission rate of CO_2, including exogenous factors like demand, supply, fuel and emission allowances.

2 Clean Spread Options

The value of a coal or gas fired power plant can be quantified with the help of clean dark spread options (coal) or clean spark spread options (gas). The spread determines the costs of transforming fuel into electricity at a given power plant efficiency. Clean options also take into account the costs of EUAs to offset the pollution of greenhouse gas.

In the following we define the spreads for the German market. The clean dark spread is defined as:

$$CDS = S - \tfrac{1}{e_{ff_1}} \left(K + e_1 A\right),$$

where S denotes the price of electricity in €/MWh, K the price of coal in €/MWh and A the price of an EUA in €/tCO_2. The variable e_1 determines how much greenhouse gas are emitted if one MWh of energy is produced at 100% efficiency. Corresponding to Abadie and Chamorro (2008), IPCC (2006) we set $e_1 = 0.34056$ tCO_2 / MWh. The power plant's efficiency is denoted by e_{ff_1}.

The clean spark spread is given by:

$$CSS = S - \tfrac{1}{e_{ff_2}} \left(G + e_2 A\right),$$

where G is the price of gas in €/MWh and e_2 the emission factor. According to Abadie and Chamorro (2008), IPCC (2006) the emission of natural gas is $e_2 = 0.20196$ tCO_2 / MWh. The efficiency of a gas fired power plant is denoted by e_{ff_2}.

The spread determines the economic value of a power plant. In an ideal world, with simplifying model assumptions, that there are neither delivery commitments nor restrictions on starting and minimum operating times of a power plant, it will be online whenever the spread is positive. Hence we can model its value by a call option on the spread with maturity T under the risk neutral measure $\mathbb{Q}$:

$$V_{CDS} = e^{-rT}\mathbb{E}^{\mathbb{Q}}\left(S_T - \tfrac{1}{e_{ff_1}}\left(K_T + e_1\, A_T\right)\right)^+, V_{CSS} = e^{-rT}\mathbb{E}^{\mathbb{Q}}\left(S_T - \tfrac{1}{e_{ff_2}}\left(G_T + e_2\, A_T\right)\right)^+.$$

The profit of a coal or gas fired power plant with a capacity of one MWh in the period $[0, T]$ can be determined by summing up the option values

$$PPV_{CDS} = \sum_{t=1}^{T} V_{CDS_t},$$
$$PPV_{CSS} = \sum_{t=1}^{T} V_{CSS_t}.$$

Figure 2.1 shows the peak hour[1] spreads in the German electricity market during the year 2012. All data were taken from the *European Energy Exchange* (EEX). Since the coal price is quoted in USD / t, we transform it from t to MWh under the assumption that hard coal burns at a rate of 7.00126 MWh / t as reported by Heizung-Direkt (2013). With the help of the exchange rate from USD to €, we receive the desired quotation. The gas price is already quoted in €/ MWh and no further transformation is necessary. Corresponding to a study by the VDI (2007) we choose the typical efficiency of a coal fired plant as $e_{ff_1} = 41.425\%$ and of a gas fired one as $e_{ff_2} = 50.625\%$. The clean dark spread was always higher than the clean spark spread. This indicates that the average coal fired power plant was more profitable than the average gas fired power plant in 2012. In fact, the clean spark spread was negative most of the time and therefore, it was not profitable to produce energy with a gas fired plant. The historic spreads also reveal a strong positive correlation to the price of electricity.

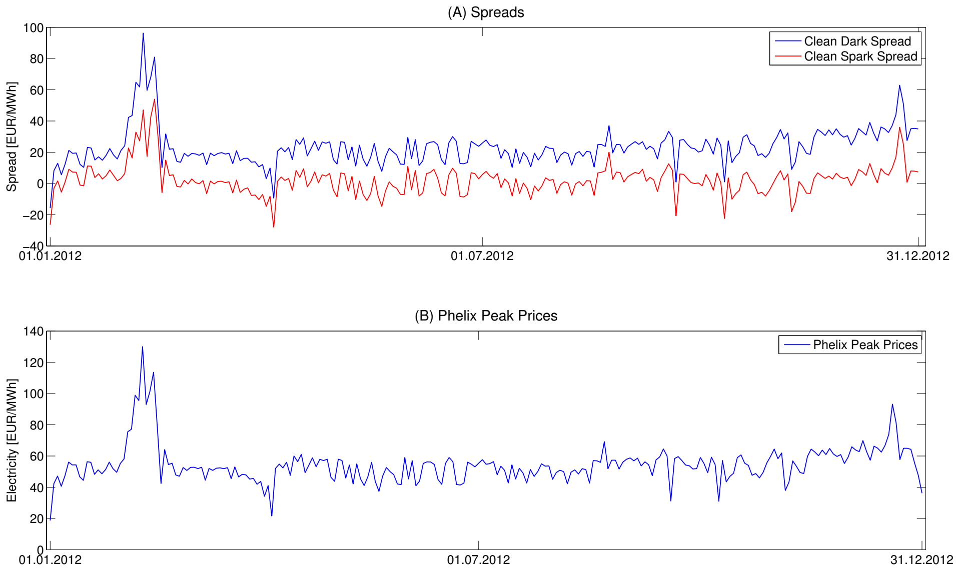

Figure 2.1: Spreads in the German electricity spot market (Peak hours)

3 The Electricity Spot Price

Since the liberalization of energy markets gathered pace, the number of modelling approaches has grown rapidly. They range from full equilibrium models to reduced form models. Equilibrium models try to identify all necessary parameters and economic fundamentals, which influence the price of energy. Therefore a huge amount of data and knowledge about all generating units, maintaining schedules, transmission constraints etc. is essential. On the one hand, these models provide deep insights into market mechanisms, but on the other hand, the tremendous amount of required data and its complexity make them unsuitable for derivatives pricing. For more details on equilibrium models we refer to Eydeland and Wolyniec (2002), Bessembinder and Lemmon (2002). Reduced form models approximate the energy price directly through stochastic processes. They

[1] trading hours 9 to 20

are easy to use for straightforward derivatives pricing, but do not incorporate any fundamental factors. The most advanced models of this group include regime switching and jumps, in Culot et al. (2013) and DeJong (2006). Structural models try to combine the advantages of the two other approaches while avoiding their disadvantages. They focus on identifying and modelling the main price drivers (e.g. demand, supply, price of resources) and derive the energy price from them.

Howison and Coulon (2009) used a joint distribution function to extract bid curves by generator type (coal, gas) from historical bid data. They were able to show a mapping between densities' parameters and fundamental drivers, such as fuel prices. In the following section we will briefly review their approach.

3.1 Stochastic Bid Curves and the Price for Electricity

We consider a market at time t with a demand D_t for energy and a capacity C_t in MWh. During day ahead auctions energy companies place m bids, consisting of quantity q_j and price p_j for $j = 1, \cdots, m$. These bids are arranged in merit order and the market operator calls upon generators until the current demand is met. Let the distribution function $F_i(S_t)$ denote the proportion of bids below S_t €/MWh for generators of fuel type $i = 1, \cdots, n$, then the spot price S_t solves the equation

$$F(S_t) = \sum_{i=1}^{n} w_i F_i(S_t) = \tfrac{D_t}{C_t}, \text{ where } \sum_{i=1}^{n} w_i = 1.$$

Hence the electricity spot price can be expressed by

$$S_t = B(\tfrac{D_t}{C_t}) := F(.)^{-1}(\tfrac{D_t}{C_t}),$$

where $B : [0,1] \to \mathbb{R}$. Coulon and Howison suggest to use logistic distributions. To improve the goodness of fit they use a trunctated domain with fixed lower and upper bounds b_L, b_U, where $b_L < \frac{D_t}{C_t} < b_U$ must hold. The demand and capacity can then be rescaled by

$$\hat{D}_t := D_t - b_L\, C_t, \qquad \hat{C}_t := (b_U - b_L)\, C_t.$$

Hence the spot price fulfills

$$F(S_t) = \sum_{i=1}^{n} w_i F_i(S_t) = \tfrac{\hat{D}_t}{\hat{C}_t}. \tag{3.1}$$

3.2 The German Spot Market

The German electricity market is relatively unique in the world. Since the federal government passed the Renewable Energy Law (Erneuerbare Energiengesetz, EEG), the capacity of sustainable generators has grown steadily. The Federal Ministry for the Environment, Nature Conservation and Nuclear Safety BMU (2004) estimates that renewable energy will represent 55% of the installed capacity in 2050. Today, solar and wind power are the most important contributors to green energy. However their production is significantly determined by the actual weather conditions. The combination of the great share of green power in the German energy mix and volatile production capacities makes the German market rather complicated. More information can be found in Hendricks (2013).

In the following we want to analyse the demand and supply situation in the German market. We will use Phelix[2] price data from the two last years. It is traded at the EPEX Spot and published as Phelix Base and Phelix Peak:

- EPEXSpot (2012) **Phelix Day Base** is the average price of the hours 1 to 24 for electricity traded on the spot market. It is calculated for all calendar days of the year as the simple average of the auction prices for the hours 1 to 24 in the market area Germany disregarding power transmission bottlenecks.

[2]Physical Electricity Index (Germany/Austria)

- EPEXSpot (2012) **Phelix Day Peak** is the average price of the hours 9 to 20 for electricity traded on the spot market. It is calculated for all calendar days of the year as the simple average of the auction prices for the hours 9 to 20 in the market area Germany disregarding power transmission bottlenecks.

During daily day ahead auctions bids ranging from -3000 € to 3000 € for each individual hour can be made. All bids are sorted by price and aggregate bids form the bid curve. Based on these results the energy price is calculated as the match of the bid and demand curve. In Figure 3.1 we see aggregate bid curves in the Phelix Energy market. In the German/Austrian market huge bids at the lower bound of -3000 € can be observed. Within our time series from the years 2011 and 2012 these bids represent up to 91.24% [3] of the total capacity. This bid behaviour cannot be found in bid data of other European countries, e.g. Switzerland and France. Comparing the production forecasts [4] of wind and solar energy to the quantity of bids below -1000 € in Figure 3.2 makes clear, that these bids belong to green producers and nucelar power plants. Since the EEG forces to prefer renewable to conventional energy, their bids can be found at the lower bound of the price range. The capacity available from nuclear power plants is also assumed to be at the lower bound, because they cannot be switched on or off easily and their running costs remain rather stable, regardless whether they are online or not. Wind production forecasts seem to be higher during winter months, while solar modules provide more energy during summer. Please note that intra-day patterns have been removed by taking the mean of all hours of each day.

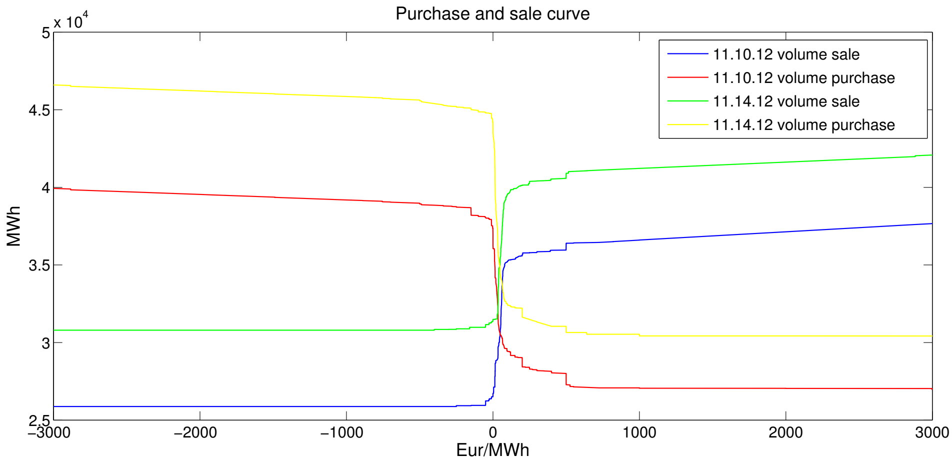

Figure 3.1: Phelix purchase and sale curve on 10th and 14th November 2012

On the contrary to the original stochastic approach by Howison and Coulon (2009), we use variable bounds to account for changing bid volumes of clean generators:

$$b_L = \frac{\sum_{i=1}^{m} q_i \mathbb{1}_{\{p_i<-1000\}}}{\sum_{i=1}^{m} q_i}, \qquad b_U = 1 - \frac{\sum_{i=1}^{m} q_i \mathbb{1}_{\{p_i>1000\}}}{\sum_{i=1}^{m} q_i}.$$

Beside renewable and nuclear energy, the German energy mix is dominated by coal (hard coal, lignite) and gas (gas, oil). After the bids of clean generators have been removed, the remaining ones belong to conventional generators. According to a study by the BDEW (2011) we set $w_1 = 0.6984$ (market capacity coal) and $w_2 = 0.3016$ (market capacity gas). In Figure 3.3 we compare the density parameters to coal and gas prices. The parameters have been estimated with Maximum Likelihood Estimation using logisitic density functions. The coal time series consists of future contracts with the nearest expiry. The gas price series is given by GASPOOL spot price data. The fitted parameters are strongly correlated to coal and gas prices. Generators move their bids in accordance to changing production costs. They also take into account the bid behaviour of other

[3] May, 29th, 2012, 12:00-13:00h

[4] Production forecasts and installed capacities are available at www.transparency.eex.com

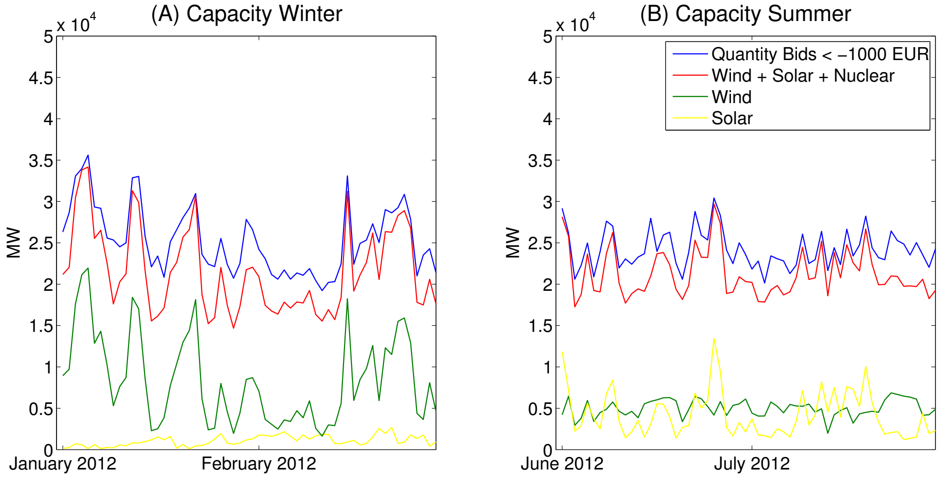

Figure 3.2: Quantity of bids below -1000 € and production forecasts

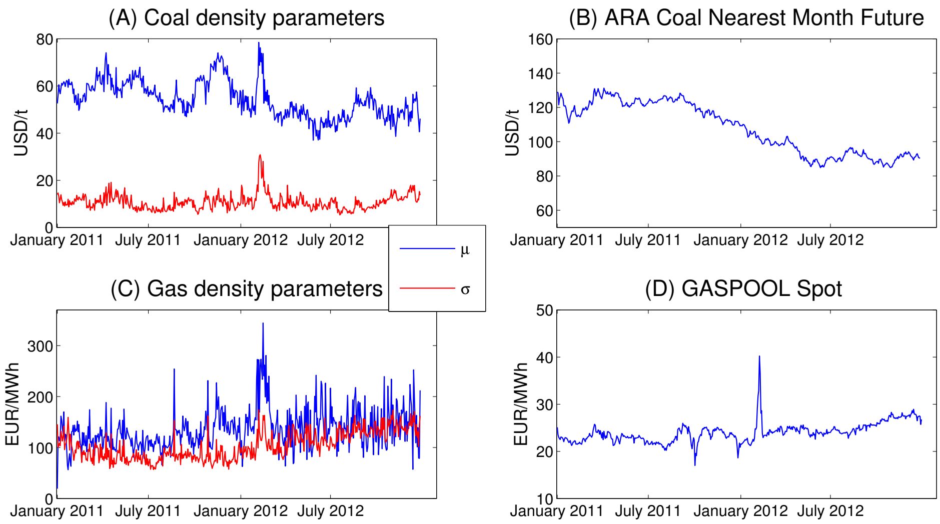

Figure 3.3: Fitted parameters and commodity prices

generators. In February 2012, for example, when the price of gas reached its peak, coal generators moved their bids upwards in order to gain an extra profit. We therefore model the parameters with the help of a linear regression model:

$$\mu_1 = \alpha_0 + \alpha_1 K + \alpha_2 G, \qquad \sigma_1 = \beta_0 + \beta_1 K + \beta_2 G,$$
$$\mu_2 = \gamma_0 + \gamma_1 K + \gamma_2 G, \qquad \sigma_2 = \delta_0 + \delta_1 K + \delta_2 G,$$

where K represents the price of coal, while G is the gas price. Table 3.1 shows the parameters, which were estimated via conditional (non-negative slopes) least square regression. The spot price for energy S_t, in the case of logistic density functions, can now be calculated by solving equation (3.1)

$$B^{-1}(.)(S_t) = w_1 \frac{1}{1+e^{-\frac{S_t-(\alpha_0+\alpha_1 K+\alpha_2 G)}{\beta_0+\beta_1 K+\beta_2 G}}} + w_2 \frac{1}{1+e^{-\frac{S_t-(\gamma_0+\gamma_1 K+\gamma_2 G)}{\delta_0+\delta_1 K+\delta_2 G}}} = \frac{\hat{D}_t}{\hat{C}_t}\,. \tag{3.2}$$

The equation possesses no analytic solution and hence must be solved numerically.

	Intercept	Slope Coal	Slope Gas
μ_1	-27.69	0.3590	1.9285
σ_1	-39.4864	0.1419	1.5298
μ_2	-52.5649	0	7.7496
σ_2	-59.1102	0	6.8472

Table 3.1: Results by conditional regression

3.3 Demand and Capacity

The demand process D_t is not directly observable in auction data. To be able to use the framework presented before, we set D_t to the point of intersection of the sale and bid curve for every hour. $\hat{D}_t$ can be calculated via the transformation $\hat{D}_t = D_t - b_{L,t}C_t$. It can be interpreted as the demand, which has to be fulfilled by conventional generators. The process $\hat{C}_t$ can be derived from C_t by $\hat{C}_t = (b_{U,t} - b_{L,t})C_t$ and understood as the capacity of conventional generators. Figure 3.4 shows that most intra-day effects of $\hat{D}_t$, $\hat{C}_t$ are compensated by the patterns of D_t, C_t and $b_{L,t}$. In order to be able to model the slight intra-day and annual pattern we model $\hat{D}_t$ as the sum of an Ornstein-Uhlenbeck process X_t and a seasonal component s_1

$$\begin{aligned} \log(\hat{D}_t) &= X_t + s_1(t) \\ dX_t &= \theta_1(\mu_1 - X_t)dt + \sigma_1 dW_t \\ s_1(t) &= \alpha_0 + \alpha_1 \cos\left(\frac{2\pi}{365 * 24}(t + \phi_1)\right) + \alpha_2 \cos\left(\frac{2\pi}{12}(t + \phi_2)\right). \end{aligned} \tag{3.3}$$

To ensure that $0 < \hat{D}_t < \hat{C}_t$ always holds, we do not simulate the capacity directly, but the margin M_t and receive $\hat{C}_t$ via $\hat{C}_t = \hat{D}_t + M_t$. M_t can be interpreted as the "unused capacity" in the market. Therefore modelling $\hat{C}_t$ reduces to modelling a strictly positive process M_t. The process $\log(M_t)$ is

$$\begin{aligned} \log(M_t) &= Z_t + s_2(t) \\ dZ_t &= \theta_2(\mu_2 - Z_t)dt + \sigma_2 d\tilde{W}_t \\ s_2(t) &= \beta_0 + \beta_1 \cos\left(\frac{2\pi}{365 * 24}(t + \omega_1)\right) + \beta_2 \cos\left(\frac{2\pi}{24}(t + \omega_2)\right). \end{aligned} \tag{3.4}$$

Figure 3.4 shows the averaged demand, capacity and margin time series. The unscaled demand process reaches its daily peak during midday, while the scaled demand process exhibits one peak around 8 am and another one around 7 pm. We therefore use $\frac{2\pi}{12}$ as the period to model the intra-day pattern. Peaks in the morning and the early evening can also often be observed in intra-day Phelix spot data. The capacity shows the same patterns as the demand process. This may be explained by power plant maintaining schedules, which are designed to meet low demand times, so that most capacity is available when demand is at a high level. The scaled capacity process is highest in the night and reaches its low at midday. The margin process is highest during the night hours, when demand reaches its lows. Throughout the year a moderate annual pattern can be seen. Demand, capacity and margin are highest during autumn/winter and lowest in spring and summer.

α_0	α_1	α_2	ϕ_1	ϕ_2
7.9544	0.0147	-0.2204	0.6520	-1.6143
β_0	β_1	β_2	ω_1	ω_2
8.6685	0.0488	0.3193	1.8095	-1.5501

Table 3.2: Seasonal parameters $\log(\hat{D}_t)$, $\log(M_t)$, cf. (3.3),(3.4)

The parameters in Table 3.2 have been estimated via least squares regression and the ones in Table 3.3 with Maximum Likelihood Estimation.

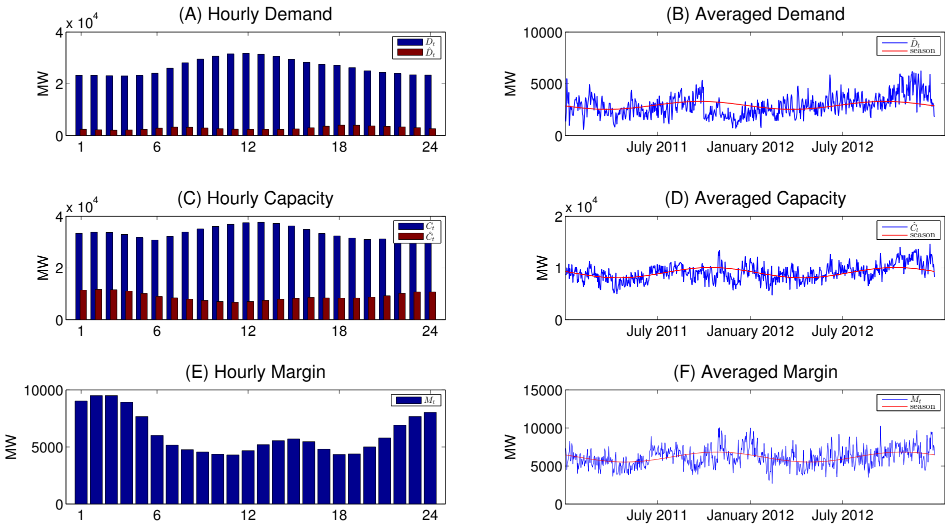

Figure 3.4: Hourly and daily demand, capacity and margin

i	θ_i	μ_i	σ_i
1	0.1304	-0.0017	0.2729
2	0.1641	-0.0007	0.2065

Table 3.3: Parameters X_t, Z_t, cf. (3.3),(3.4)

3.4 Gas and Coal

The fuel processes K_t, G_t are modelled with two correlated Ornstein-Uhlenbeck processes. In our data set no clear seasonal pattern can be observed. So we drop a seasonal component.

$$\begin{aligned} \log(K_t) &= Q_{1,t}, \\ dQ_{1,t} &= \theta_{Q_1}(\mu_{Q_2} - Q_{1,t})dt + \sigma_{Q_1} dW_{1,t}, \quad Q_{1,0} = q_{1,0} \in \mathbb{R}_+, \\ \log(G_t) &= Q_{2,t}, \\ dQ_{2,t} &= \theta_{Q_2}(\mu_{Q_2} - Q_{2,t})dt + \sigma_{Q_2} dW_{2,t}, \quad Q_{2,0} = q_{2,0} \in \mathbb{R}_+, \end{aligned} \tag{3.5}$$

where $dW_{1,t}dW_{2,t} = \rho dt$. In our empiric time series we have observed a correlation of $\rho = 0.0872$. Table 3.4 shows the estimated parameters of the logarithmic fuel time series.

i	θ_{Q_i}	μ_{Q_i}	σ_{Q_i}
1	0.00004	4.8598	0.0021
2	0.0026	3.219	0.007

Table 3.4: Parameters $Q_{1,t}$, $Q_{2,t}$, cf. (3.5)

3.5 Transformation

We used the logarithm of prices in (3.3) - (3.5). As we want to price derivatives with them, we have to transform them to Itô processes. We will do this for the demand and coal process. The application to the other ones is straight forward. Using (3.3) and the ansatz $\hat{D}_t := f(t, X_t) =$

$e^{X_t+s_1(t)}$ we apply *Itô's Lemma* and obtain

$$\begin{aligned} d\hat{D}_t &= \left(e^{X_t+s_1(t)}\theta(\mu_1 - X_t) + s_1'(t)e^{X_t-s_1(t)} + \frac{1}{2}e^{X_t+s_1(t)}\sigma_1^2\right) dt + e^{X_t+s_1(t)}\sigma_1\, dW_t \\ &= \left(\theta\left[\mu_1 - \log(\hat{D}_t) + s_1(t)\right] + s_1'(t) + \frac{1}{2}\sigma_1^2\right)\hat{D}_t\, dt + \hat{D}_t\sigma_1 dW_t. \end{aligned} \tag{3.6}$$

For (3.5) we use $K_t := f(t, Q_t) = e^{Q_t}$ and *Itô's Lemma* yields

$$\begin{aligned} dK_t &= \left(e^{Q_{1,t}}\theta_{Q_1}\left[\mu_{Q_1} - Q_t\right] + \frac{1}{2}e^{Q_{1,t}}\sigma_{Q_1}^2\right) dt + e^{Q_{1,t}}\sigma_Q\, dW_t \\ &= \left(\theta_{Q_1}\left[\mu_{Q_1} - \log(K_t)\right] + \frac{1}{2}\sigma_{Q_1}^2\right) K_t\, dt + K_t\sigma_{Q_1}\, dW_t. \end{aligned} \tag{3.7}$$

4 European Emission Trading System

The *European Union Emission Trading System* (EU ETS) was launched in 2005 to reduce the emission of greenhouse gas. The 31 participants (27 EU member states and Croatia, Iceland, Norway, Liechtenstein) agreed on a reduction of 21% of CO_2 emissions until 2020 compared to 2005. The system covers over 11,000 installations in the energy sector and electricity intense industries, such as steel, paper, cement industries etc. In total, about 45% of all EU emissions are limited by the EU ETS Commission (January 2013).

The EU ETS is based on the so-called *cap and trade* principle. The cap determines the total amount of greenhouse gas that can be emitted by all installations. After each year a company has to surrender enough *European Union Emission Allowances* (EUA) to cover their emissions. If more allowances are needed or if some remain unused, they can be freely traded among market participants. Hence, the emission reduction is performed where it is economically most senseful at lowest possible costs. If a company does not have enough EUAs to offset its pollution, a penalty of 100 € is charged. In addition, the allowance has to be handed in later, e.g. by reducing emissions in the next year or by buying it.

Due to the rapidly evolving emission markets, the amount of modelling approaches is steadily growing. Benz and Trück (2009) and Daskalakis et al. (2009) discuss a wide class of reduced form models, such as regime switching and models including jumps, which are common in computational finance, to approximate the CO_2 price dynamics of the EU ETS. As we want to link fundamental factors, such as fuel prices, to the price of carbon, we introduce a structural approach. It is closely connected to the one by Carmona et al. (2012), but we will deviate from their approach by modelling the CO_2 emission rate of the German market with the help of the framework of the previous chapters.

4.1 The CO_2 Emission Rate

The market emission rate μ_e is a positive and bounded function, which shall be influenced by the actual demand for energy, the available capacity and coal and gas prices. Furthermore, the emission rate shall react on changes in CO_2 allowance prices, which is basically the aim of the EU ETS. If carbon prices fluctuate, we expect a change in the merit order and hence a varying emission of greenhouse gas. To receive a feedback of EUA prices, we assume the electricity spot price to react towards changing prices. Due to the fact that generators have to buy certificates to offset their pollution and that they at least proceed any changes in production costs partially to the market, this assumption seems to be rather relevant. Nevertheless we could not find any empirical evidence for a linear relationship of allowance prices towards the density parameters within our data set. This may be explained by the low prices of CO_2 allowances during the years 2011 and 2012. In the sequel we assume a linear dependency of the form below to exist for the German market with the fuels coal and gas:

$$\begin{aligned} \mu_1 &= \alpha_0 + \alpha_1\, K + \alpha_2\, G + \alpha_3\, A, & \sigma_1 &= \beta_0 + \beta_1\, K + \beta_2\, G + \beta_3\, A, \\ \mu_2 &= \gamma_0 + \gamma_1\, K + \gamma_2\, G + \gamma_3\, A, & \sigma_2 &= \delta_0 + \delta_1\, K + \delta_2\, G + \delta_3\, A. \end{aligned}$$

Furthermore, we assume that the emission costs are completely passed to the market. We therefore choose the sensitivities corresponding to Abadie and Chamorro (2008), IPCC (2006), VDI (2007):

$$\alpha_3 = \frac{e_1}{e_{ff_1}} = \frac{0.34056}{0.41425} \frac{tCO_2}{MWh}, \qquad \gamma_3 = \frac{e_2}{e_{ff_2}} = \frac{0.20196}{0.50625} \frac{tCO_2}{MWh}.$$

The parameter choice for $\beta_3 = \frac{\alpha_3}{10}$ and $\delta_3 = \frac{\delta_3}{10}$ is arbitrary. Since the intercept has been calibrated without any influence of carbon, we reduce it by the allowance sensitivity multiplied with the observed mean of EUAs of 10.3268€ / tCO_2. To be able to receive the current emission rate for the German market, we have to calculate how much energy is supplied by dirty generators. The percentage of clean energy is given by b_L. After these bids have been removed from the data set, $\hat{D}_t$ is the remaining demand which has to be fulfilled by conventional generators. With the help of model (3.1) and (3.2) respectively, we can calculate the percentage of the remaining demand, which is satisfied by generators of fuel $i = 1, 2$ via $w_i F_i(S_t) = w_i F_i(B(\frac{\hat{D}_t}{\hat{C}_t}))$. Their contribution towards demand $\hat{D}_t$ is then given by

$$C_t^i := w_i F_i(S_t) \hat{C}_t = w_i F_i \left(B \left(\frac{\hat{D}_t}{\hat{C}_t} \right) \right) \hat{C}_t$$

for $i = 1, 2$. If we assume that each type of generator has its specific emission rate $\hat{e}_i$ (tCO_2 / MWh), the overall emission rate is then

$$\mu_e(\hat{D}_t, A_t, M_t, K_t, G_t) = \hat{e}_1 w_1 F_1(S_t) \underbrace{(\hat{D}_t + M_t)}_{\hat{C}_t} + \hat{e}_2 w_2 F_2(S_t) \underbrace{(\hat{D}_t + M_t)}_{\hat{C}_t}.$$

In the sequel we will set $\hat{e}_1 = \frac{0.34056}{0.41425} \frac{tCO_2}{MWh}$, $\hat{e}_2 = \frac{0.20196}{0.50625} \frac{tCO_2}{MWh}$. The complexity of our emission rate can be reduced, if the margin process M_t is chosen to be deterministic. Thus we replace it by its seasonal, deterministic component $e^{s(t)}$ and write $\mu_e(\hat{D}_t, A_t, K_t, G_t) := \mu_e(\hat{D}_t, A_t, e^{s(t)}, K_t, G_t)$.

In Figure 4.1 (A) we see the emission rate for changing fuel prices. High coal prices lead to an increased usage of gas fired power plants and thus to less pollution. Contrarily if gas is expensive, more coal power plants will be used and hence there is a higher emission of greenhouse gas. In (B) we analyse the influence of CO_2 allowances on the emission rate. If the capacity utilisation is low, gas plants can provide enough electricity to fulfill the complete demand and we see a reduction of 4.92% in emission. In the case of a high capacity utilisation dirty coal power plants are needed to fulfill demand, since there is not enough capacity from clean gas plants. Here the decrease in emitted CO_2 is only 2.57%. This emission reduction seems quite low, since it does not incorporate any changes in the total capacities of coal and gas generators. It can be interpreted as the short term effect of the emission trading system. In the long run we expect energy companies to change their power plant portfolio in favour of cleaner plants, if allowances remain at a high level. This results in a higher reduction of greenhouse gas emissions. Therefore, the model parameters have to be recalibrated if the market environment alters.

4.2 Emission Allowances

In this section we want to price emission allowances in a risk-neutral setting. To avoid any difficulties resulting from estimating the market risk premium, we assume it to be zero for the sake of simplicity. Hence, the calibrated processes under the real world measure $\mathbb{P}$ remain the same under the risk neutral measure $\mathbb{Q}$.

The cumulative emission can be gained by integrating the emission rate over time

$$E_t = \int_0^t \mu_e(\hat{D}_s, A_s, K_s, G_s) ds.$$

Thus we have

$$dE_t = \mu_e(\hat{D}_t, A_t, K_t, G_t) dt.$$

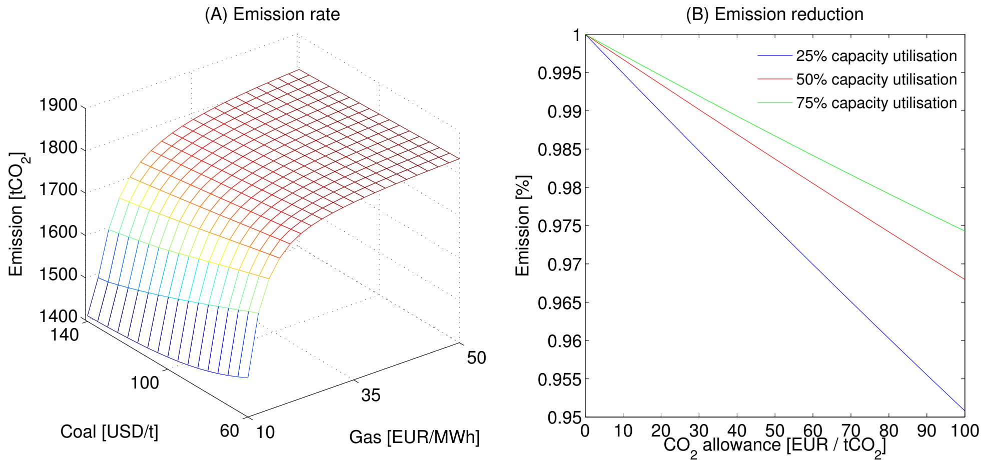

Figure 4.1: (A): μ_e for D=2500, M=2500, A=25
(B): Influence of allowances on the emission rate, $K = 100$, $G = 25$

In combination with the underlying stochastic processes, we receive a four dimensional system of *stochastic differential equations* (SDE):

$$\begin{cases} d\hat{D}_t = \mu_{\hat{D}}(\hat{D}_t)dt + \sigma_{\hat{D}}(\hat{D}_t)dW_t, & \hat{D}_0 = \hat{d}_0 \in \mathbb{R}_+, \\ dK_t = \mu_K(K_t)dt + \sigma_K(K_t)d\hat{W}_{1,t}, & K_0 = k_0 \in \mathbb{R}_+, \\ dG_t = \mu_G(G_t)dt + \sigma_G(G_t)d\hat{W}_{2,t}, & G_0 = g_0 \in \mathbb{R}_+, \\ dE_t = \mu_e(\hat{D}_t, A_t, K_t, G_t)dt, & E_0 = 0. \end{cases}$$

If we consider the value of an EUA A as a derivative of the underlying processes, we can obtain with the help of *Itô's Lemma*

$$\begin{aligned} dA = \Bigg[& \frac{\partial A}{\partial t} + \mu_{\hat{D}}(\hat{D}_t)\frac{\partial A}{\partial \hat{D}_t} + \mu_e(\hat{D}_t, A_t, K_t, G_t)\frac{\partial A}{\partial E_t} \\ & + \mu_K(K_t)\frac{\partial A}{\partial G_t} + \mu_G(G_t)\frac{\partial A}{\partial G_t} + \frac{1}{2}\sigma_{\hat{D}}(\hat{D}_t)^2\frac{\partial^2 A}{\partial \hat{D}_t^2} \\ & + \frac{1}{2}\sigma_K(K_t)^2\frac{\partial^2 A}{\partial K_t^2} + \frac{1}{2}\sigma_G(G_t)^2\frac{\partial^2 A}{\partial G_t^2} + \rho\sigma_K(K_t)\sigma_G(G_t)\frac{\partial^2 A}{\partial K_t \partial G_t} \Bigg] dt \\ & + \sigma_{\hat{D}}(\hat{D}_t)\frac{\partial A}{\partial \hat{D}_t}dW_t + \sigma_K(K_t)\frac{\partial A}{\partial K_t}d\hat{W}_{1,t} + \sigma_G(G_t)\frac{\partial A}{\partial G_t}d\hat{W}_{2,t}, \end{aligned}$$

where we have simplified our notation by $A = A(t, \hat{D}_t, E_t, K_t, G_t)$. Standard risk-neutrality arguments with risk free interest rate r, yield the *partial differential equation* (PDE)

$$\begin{aligned} \frac{\partial A}{\partial t} + \mu_{\hat{D}}(\hat{D})\frac{\partial A}{\partial \hat{D}} + \mu_e(\hat{D}, A, K, G)\frac{\partial A}{\partial E} + \mu_K(K)\frac{\partial A}{\partial K} + \mu_G(G)\frac{\partial A}{\partial G} & \\ + \frac{1}{2}\sigma_{\hat{D}}(\hat{D})^2\frac{\partial^2 A}{\partial \hat{D}^2} + \frac{1}{2}\sigma_K(K)^2\frac{\partial^2 A}{\partial K^2} + \frac{1}{2}\sigma_G(G)^2\frac{\partial^2 A}{\partial G^2} & \\ + \rho\sigma_K(K)\sigma_G(G)\frac{\partial^2 A}{\partial K \partial G} - rA = 0, & \end{aligned} \tag{4.1}$$

At maturity T one either has to pay the penalty π if cumulative emissions E exceed the emission cap E_{cap} or nothing if emissions have not reached the cap. So the allowance's payoff is given by

$$A(T, D, E, K, G) = \pi \mathbb{1}_{[E_{cap}, \infty)}(E). \tag{4.2}$$

A discussion of boundary conditions, needed to specify the solution, has been moved to Section 6.

In Figure 4.2 we see the impact of the emission cap on the value of CO_2 allowances with fixed fuel prices. If the cap is low and likely to be reached, the right to emit greenhouse gas is expensive.

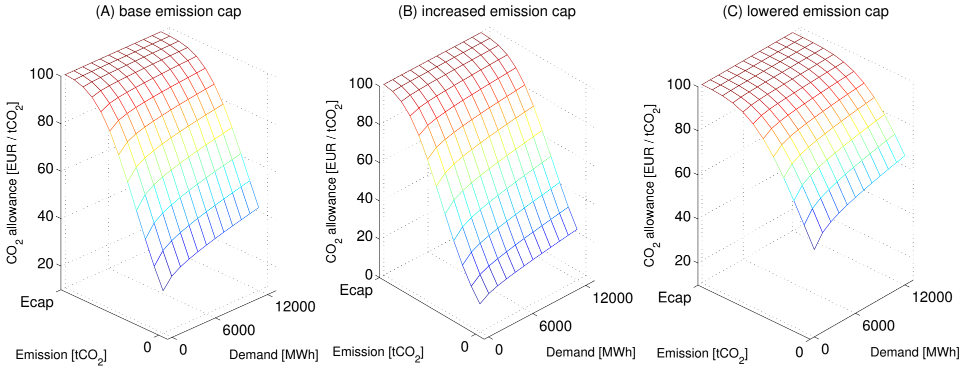

Figure 4.2: CO_2 allowance with base emission cap, emission cap +20%, emission cap -20% in direction of demand and emission

Vice versa, it is cheaper if the cap is high. In Figure 4.3 the allowance value in direction of both fuel processes for fixed values in demand and emission is plotted. We see that the value exhibits the same form as the emission rate. If emission is high, there is a higher chance that the emission cap is reached and hence the certificate's value rises.

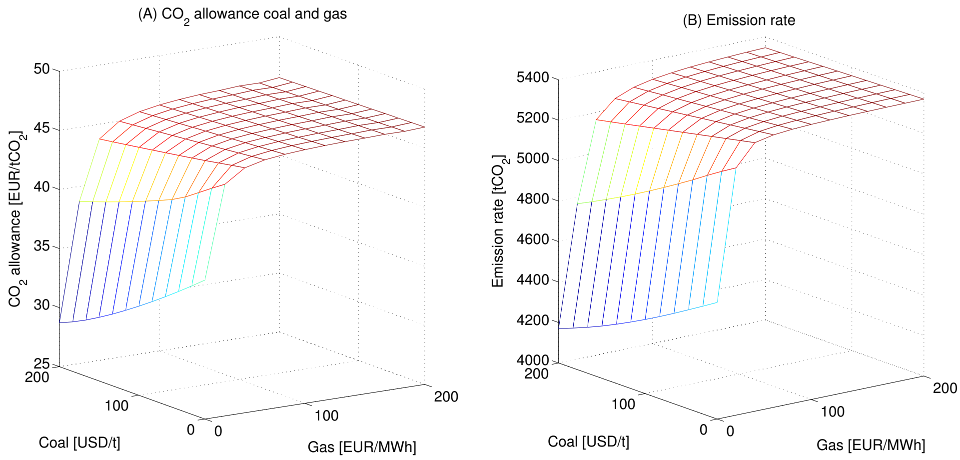

Figure 4.3: CO_2 allowance and emission in direction of coal and gas

5 Case Studies

The spread between the price for electricity and its production costs is influenced by many factors. Power plants characteristics, such as its efficiency, have a great impact. But also external factors, like the price of EUAs and fuel costs, can lower or widen the spread significantly. While we expect clean power plants to benefit from higher EUA prices, we expect inefficient and dirty plants to have a greater economic value if fuel and the right to emit greenhouse gas is cheap.

In the past years a coal fired power plant was much more profitable than a cleaner gas fired power plant (cf. 2.1). CO_2 allowance prices were steadily declining in recent history and critics argue, that the EU ETS has failed. In the following we want to consider and simulate several future scenarios. We want to investigate how changes in the key parameters influence the spreads and therefore the value of a power plant.

Our base scenario, Tables 5.1 and 5.2, considers a market with an emission cap of 20 million t / CO_2, which results in an allowance price of $A_0 = 33.62$ € (with $K_0 = 100$, $G_0 = 25$, $\hat{D}_0 = 2300$). The coal starting price is 100 USD / t and the gas price 25 € / MWh. For the sake of simplicity we assume a fixed exchange rate 1.30 USD = 1 € to convert the coal price into euro. The market is simulated for 8760 trading hours or equivalently one year. If the emission cap is exceeded at the end of the trading period, a penalty of 100 € is due. The spread options V_{CDS_t} and V_{CSS_t} are computed for maturities $t = 1, \cdots, 8760$. To visualise our results and gain insights into the intra-day profitability of a power plant, we average option prices for all 24 trading hours of a day. Since there is no strong annual pattern in the German electricity market, we omit it in our analysis.

$\hat{D}_0$	K_0	G_0
2300	100	25

Table 5.1: Parameters base scenario

T	π	E_{cap}	r
8760	100	$20 * 10^6$	0.02

Table 5.2: Parameters EU ETS

5.1 Influence of the Emission Cap

We are now interested in the influence of the emission cap on the spread of coal and gas fired power plants. When the emission cap is very high and not likely to be reached, we expect low EUA prices and high clean dark options. On the contrary we expect higher clean spark spread options if the emission cap is low. In the following we will compare the base scenario to a market with a generous emission cap $E_{cap} = 24 * 10^6$ t CO_2 (Base + 20%) and to a market with a restrictive cap of $E_{cap} = 16 * 10^6$ t CO_2 (Base - 20%). In the first case an allowance starting price of $A_0 = 13.73$ can be observed, while the latter leads to $A_0 = 74.30$.

In Figure 5.1 we compare the averaged spread option values for normally, lowly and highly efficient plants. The intra-day pattern with a top in the morning and another one in the evening corresponds to the demand cycle $\hat{D}$. The increase in demand leads to a higher price for electricity and thus to higher spreads. The value of a coal power plant shows adverse effects towards high allowance prices. Especially lowly efficient plants suffer from high prices, while highly efficient plants do not seem to be influenced. During periods of high demand, when the price for electricity is near the top of the stack, these cleaner power plants gain an extra profit by a price, which was set by less efficient coal plants. Hence, the negative effect of high carbon prices is compensated by an extra profit. Gas fired power plants are strongly influenced by the prices of emission allowances. If the cap is restrictive and hence the right to emit greenhouse gas is expensive, gas power plants are pushed in front of coal generators in the merit order. On the contrary they are behind coal generators in the merit order if allowances are cheap.

The simulation reveals that a reduction of greenhouse gas can be achieved if the emission cap is likely to be reached. Thus, clean gas plants are favoured over coal plants and especially over lowly efficient and dirty coal plants.

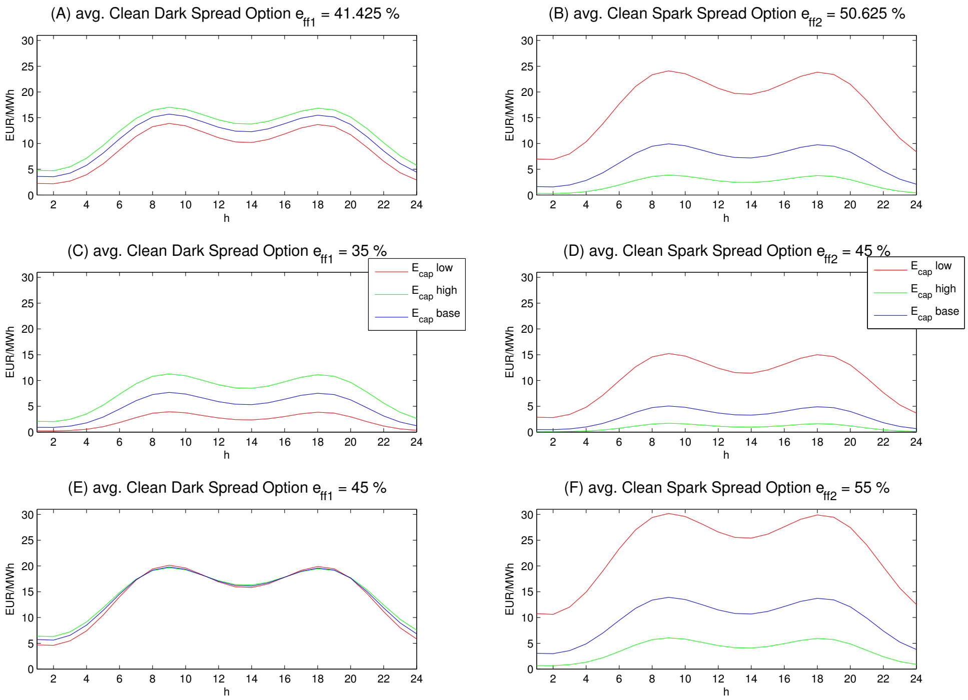

Figure 5.1: Influence of the emission cap

5.2 Influence of the Demand Process

In the second trading phase of the EU ETS, the European Union struggled into an economic recession. The lowered need for electricity led to a steady decline of emission certificates. In this case study, we want to simulate the effects of an economic boom and a recession. The boom will be modelled by a demand process, which is increased by 5% to simulate industrial prosperity and a strong need for energy. Vice versa, the depression is modelled by a drop in demand of 5%.

In Figure 5.2 we compare averaged clean spread options in the recession and boom scenarios. The higher need for energy in the boom scenario leads to a higher overall emission of greenhouse gas, hence, the allowances are more expensive. The starting values for EUAs are $A_0^{base} = 33.62$, $A_0^{+5\%} = 43.48$ and $A_0^{-5\%} = 24.86$. The boom scenario favours gas fired power plants in two points: the higher demand for energy pushes the price for electricity upwards and additionally carbon prices rise. Hence, we can see a strong impact of demand shifts on gas plants. In the case of coal plants rising EUAs have a negative effect. The plots reveal that the positive effects of increasing electricity prices on the one hand and adverse effects of rising EUAs on the other hand almost completely equate each other. In the case of lowly efficient coal plants, there is hardly any change in the spread options. The higher (lower) electricity price is almost completely compensated by higher (lower) carbon dioxide allowances.

5.3 Influence of Changes in initial Fuel Prices

We now want to simulate the influence of coal and gas prices on spread options. Since the fuel processes are mean reverting, this corresponds to a temporary shift in fuel prices.

In the case of gas, we consider starting values of $G_0 = 15$€ / MWh and $G_0 = 35$€ / MWh. The mean reversion rate of $\theta_2 = 0.0026$ pushes the price back to its mean within approximately four months. In Figure 5.3 we see the spreads of normally, lowly and highly efficient plants with different gas starting values. When we calibrated our model to EPEX data in Section 3.2, we

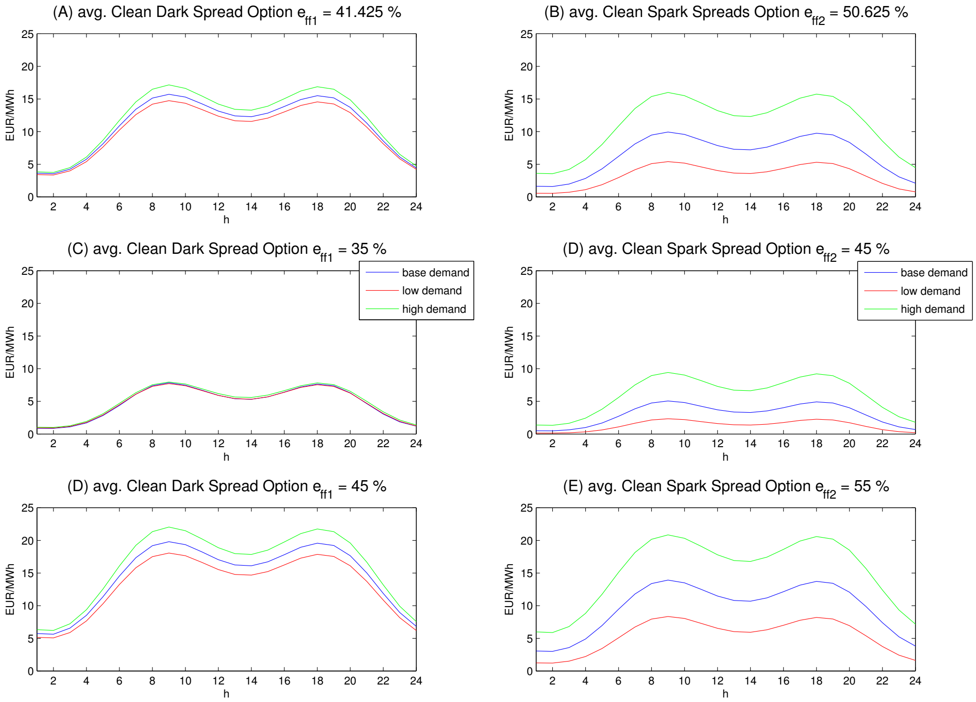

Figure 5.2: Influence of the demand

have seen, that coal plant operators take the actual gas price into account. They move their bids according to the gas price to ensure that they are in front of gas plants in the merit order. Hence if gas prices are low, the electricity spot price will be low too. Therefore gas plants do not profit from a declining gas price as we would have intuitively expected. Contrarily if gas prices are high, coal generators move their mean bid level upwards to gain an extra profit.

In Figure 5.4 we compute the spreads with coal starting values $K_0 = 80$ USD / t and $K_0 = 120$ USD / t. Coal plant operators move their bid levels with a slope of 0.359[5], while the production costs of an average plant move by 0.3448[6], if the price of coal changes by one USD. Hence their margin remains rather stable if the coal price fluctuates. Although, gas plants are behind coal plants at our simulated price levels, they gain an extra profit during peak times in demand, if coal and thus the price for electricity is high. Please note, that compared to Figure 5.3, the effects of changes in the initial coal price are stronger than for the gas price, since the coal process is less mean reverting.

[5] Please compare to Table 3.1

[6] Assuming coal burns at a rate of $7.00126 \frac{MWh}{t}$ and $e_{ff_1} = 41.425\%$: $\frac{1}{7.00126} \frac{1}{0.41425} = 0.3448$

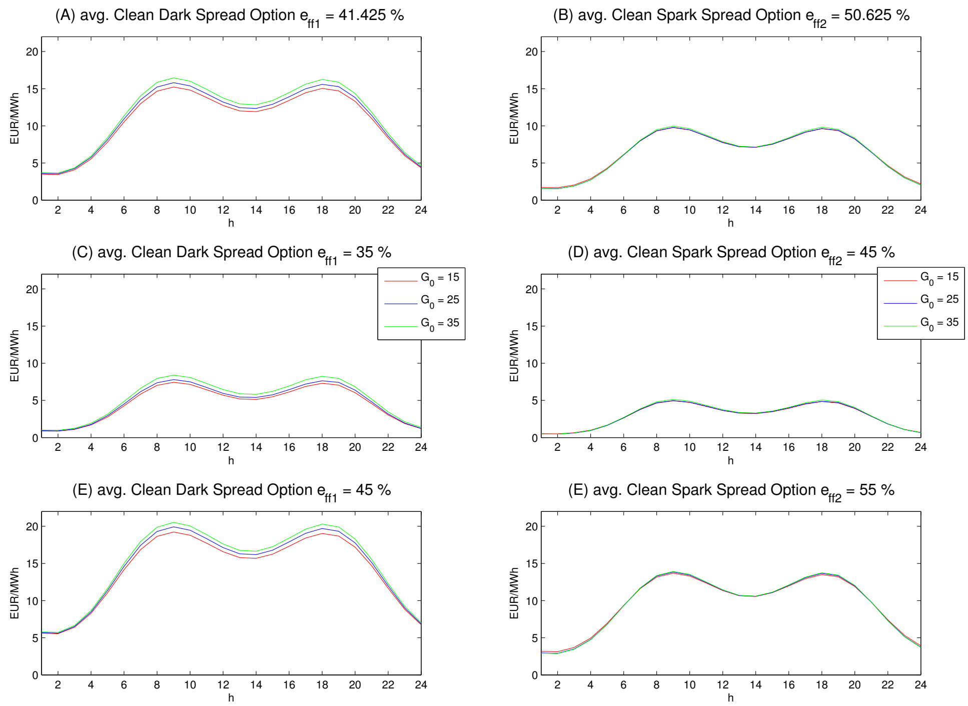

Figure 5.3: Influence of gas prices (base scenario)

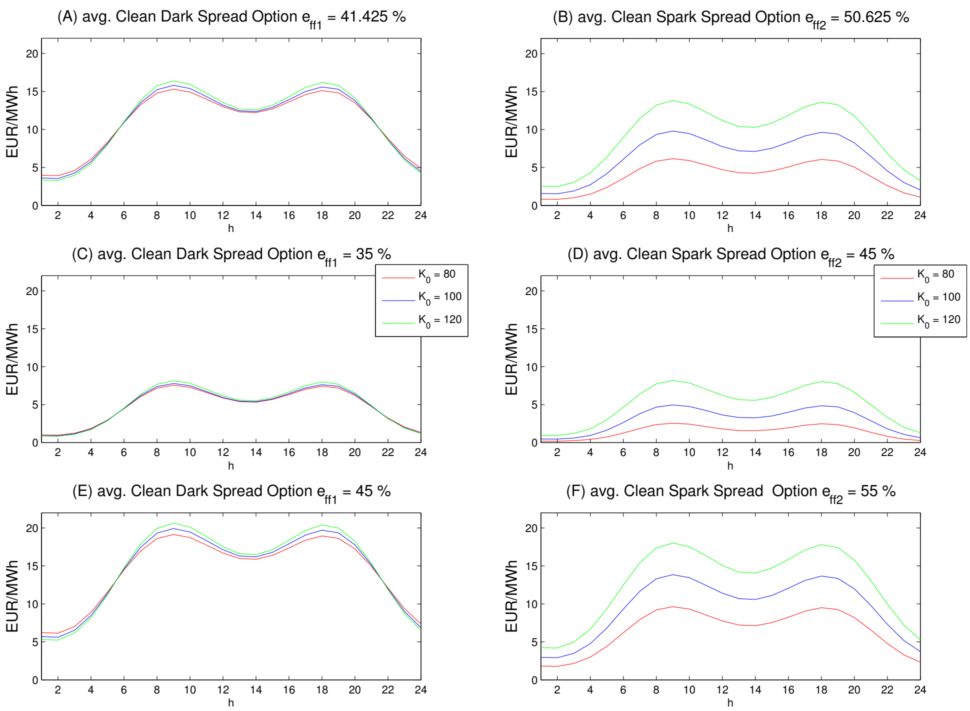

Figure 5.4: Influence of coal prices (base scenario)

6 Numerical Simulation

In this section we want to present numerical methods to solve the PDE (4.1) and to compute clean spread options.

6.1 Numerical Solution of EUAs

The PDE we want to solve numerically is given by (4.1)

$$\begin{aligned}\frac{\partial A}{\partial t} + \mu_d(t,D)\frac{\partial A}{\partial D} + \frac{1}{2}\sigma_d(D)^2\frac{\partial^2 A}{\partial D^2} + \mu_k(K)\frac{\partial A}{\partial K} + \frac{1}{2}\sigma_k(K)^2\frac{\partial^2}{\partial K^2} + \mu_g(G)\frac{\partial A}{\partial G} \\ + \frac{1}{2}\sigma_g(G)^2\frac{\partial^2 A}{\partial G^2} + \rho\sigma_k(K)\sigma_g(G)\frac{\partial^2 A}{\partial K \partial G} + \mu_e(D,A,K,G)\frac{\partial A}{\partial E} - rA = 0\end{aligned}$$

with terminal condition at maturity $t = T$

$$A(T,D,E,K,G) = \pi \mathbb{1}_{[E_{cap},\infty)}(E).$$

The PDE shall be solved on the set $\Omega = (0, D_{max}] \times (0, E_{cap}] \times (0, K_{max}] \times (0, G_{max}]$ for all $t \in [0,T]$. We use the *Fichera function* to investigate if boundary conditions are necessary. A detailed introduction into *Fichera theory* can be found in Duffy and Kienitz (2009), Kichenassamy (2007). Let $\nu = (\nu_D, \nu_E, \nu_K, \nu_G)$ be the inward normal vector to the boundary $\partial\Omega$, then the *Fichera function* is defined on the part of the boundary, where the characteristic form is zero. The function is given by

$$\begin{aligned}b = \left(\mu_d(D) - \frac{1}{2}\frac{\partial \sigma_d(D)^2}{\partial D}\right)\nu_D + \left(\mu_k(K) - \frac{1}{2}\frac{\partial \sigma_k(K)^2}{\partial K} - \frac{1}{2}\rho\sigma_g(G)\frac{\partial \sigma_k(K)}{\partial K}\right)\nu_K \\ + \left(\mu_g(G) - \frac{1}{2}\frac{\partial \sigma_g(G)^2}{\partial G} - \frac{1}{2}\rho\sigma_k(K)\frac{\partial \sigma_g(G)}{\partial G}\right)\nu_G + \mu_e(D,A,K,G)\nu_E.\end{aligned}$$

If $b < 0$, the flow of information is inward and boundary conditions need to be specified. Contrarily if $b \geq 0$, the flow is outward and no boundary conditions are allowed. At the boundary $\partial\Omega$, which corresponds to $D = 0$, $b \geq 0$ is fulfilled. At the lower boundary of both fuel processes $(K = 0, G = 0)$ the *Fichera function* is also positive. Since the emission rate μ_e is always positive and $\nu_E = -1$ at the boundary $E = E_{cap}$ the function is negative. Hence a boundary condition has to be specified. If the emission cap has already been reached, the penalty π surely has to be paid and the allowance value A is given by

$$A(t,D,E,K,G) = \pi e^{-r(T-t)} \text{ if } E \geq E_{cap}.$$

We discretise our five dimensional grid by

$$\begin{aligned} 0 < D_0 < D_1 < \cdots < D_{N_D-2} < D_{N_D-1} &= D_{max}, \\ 0 < K_0 < K_1 < \cdots < K_{N_K-2} < K_{N_K-1} &= K_{max}, \\ 0 < G_0 < G_1 < \cdots < G_{N_G-2} < G_{N_G-1} &= G_{max}, \\ 0 = E_0 < E_1 < \cdots < E_{N_E-2} < E_{N_E-1} &= E_{cap} - \Delta E, \\ 0 = t_0 < t_1 < \cdots < t_{N_T-1} < t_{N_T} &= T, \end{aligned}$$

where $\Delta D = D_{i+1} - D_i$, $\Delta E = E_{j+1} - E_j$, $\Delta K = K_{k+1} - K_k$, $\Delta G = G_{g+1} - G_g$, $\Delta t = t_{n+1} - t_n$ for all i, j, k, g, n. In the following we want to simplify our notation and write

$$\begin{aligned} A^n_{i,j,k,g} &= A(t_n, D_i, E_j, K_k, G_g), \\ \mu_{d;n,i} &= \mu_d(t_n, D_i), \\ \sigma_{d;i} &= \sigma_d(D_i), \\ \mu_{k;k} &= \mu_k(K_k), \\ \sigma_{k;k} &= \sigma_k(K_k), \\ \mu_{g;g} &= \mu_g(G_g), \\ \sigma_{g;g} &= \sigma_g(G_g), \\ \mu_{e;n,i,j,k,g} &= \mu_e(D_i, A^n_{i,j,k,g}, K_k, G_g). \end{aligned}$$

The PDE will be solved numerically with the help of *Finite Difference Methods* (FDM). As the "curse of dimensionality" shows its effects very quickly in high dimensional problems, we try to lower the computational effort compared to standard FDM schemes. Standard schemes use broadly banded matrix equations, which are very expensive to solve. Alternating Direction Implicit (ADI) schemes avoid broad banded systems by decomposing them into simpler tridiagonal matrices. These can be solved efficiently by LU-decomposition. More information can be found in Duffy (2006), Haentjens and Hout (2012). In order to simplify our notation, we will use an operator notation in our scheme:

$$\begin{aligned}
\delta_x^+ u(x) &= u(x+\Delta x) - u(x)\\
\delta_x^- u(x) &= u(x) - u(x-\Delta x)\\
\delta_x^0 u(x) &= u(x+\Delta x) - u(x-\Delta x)\\
\delta_x^2 u(x) &= u(x+\Delta x) - 2u(x) + u(x-\Delta x)\\
\delta_x^0\delta_y^0 u(x,y) &= u(x+\Delta x, y+\Delta y) - u(x-\Delta x, y+\Delta y)\\
&\quad - u(x+\Delta x, y-\Delta y) + u(x-\Delta x, y-\Delta y)
\end{aligned}$$

The scheme we use is given by

$$\begin{aligned}
\Big(1+\frac{1}{2}r\Delta t - \frac{1}{2}\mu_{d;n+\frac{1}{2},i}\frac{\Delta t}{\Delta D}\delta_D^{+/-} - \frac{1}{4}\sigma_{d,i}^2\frac{\Delta t}{\Delta D^2}\delta_D^2\Big)\Delta A^* &= \Big(-r\Delta t + \mu_{d;n+\frac{1}{2},i}\frac{\Delta t}{\Delta D}\delta_D^{+/-} + \frac{1}{2}\sigma_{d;i}^2\frac{\Delta t}{\Delta D^2}\delta_D^2\\
&\quad +\mu_{k;k}\frac{\Delta t}{2\Delta K}\delta_K^0 + \frac{1}{2}\sigma_{k;k}^2\frac{\Delta t}{\Delta K^2}\delta_K^2 + \mu_{g;g}\frac{\Delta t}{2\Delta G}\delta_G^0 + \frac{1}{2}\sigma_{g,g}^2\frac{\Delta t}{\Delta G^2}\delta_G^2\\
&\quad +\mu_{e;n+1,i,j,k,g}\frac{\Delta t}{\Delta E}\delta_E^+ + \rho\sigma_{k;k}\sigma_{g;g}\frac{\Delta t}{4\Delta K\Delta G}\delta_K^0\delta_G^0\Big)A^{n+1}\\
\Big(1-\frac{1}{2}\mu_{k;k}\frac{\Delta t}{2\Delta K}\delta_K^0 - \frac{1}{4}\sigma_{k;k}^2\frac{\Delta t}{\Delta K^2}\delta_K^2\Big)\Delta A^{**} &= \Delta A^*\\
\Big(1-\frac{1}{2}\mu_{g;g}\frac{\Delta t}{2\Delta G}\delta_G^0 - \frac{1}{4}\sigma_{g;g}^2\frac{\Delta t}{\Delta G^2}\delta_G^2\Big)\Delta A^{***} &= \Delta A^{**}\\
\Big(1-\frac{1}{2}\mu_{e;n+1,i,j,k,g}\frac{\Delta t}{\Delta E}\delta_E^+\Big)\Delta A &= \Delta A^{***}
\end{aligned} \tag{6.1}$$

with $\Delta A = A^n - A^{n+1}$. This scheme can be derived by factorizing a Crank-Nicolson scheme and rewriting the system in the so-called **delta formulation** (Thomas (1998)). According to Craig and Sneyd (1988), Haentjens and Hout (2012) the mixed derivative is treated explicitly. Please note that $n+1$ denotes the explicit part, since we are stepping backwards in time. In direction of demand we use an upwind scheme in the convective part to account for a high **Péclet number**. In direction of both fuel processes no convection dominance can be found and hence central differences are deployed to approximate the first derivative. In order to handle the non-linearity in the emission drift function, the allowance value of the previous step is used. Since the feedback of its price on the emission rate is rather moderate (cf. 4.1), we do not expect to introduce a large error. In the last leg we have to determine the boundary condition in the artificial variable ΔA. Since ΔA is the difference in an allowance value of two consecutive time steps, we choose the boundary condition as

$$\Delta A = \pi\left(e^{-r(T-t_n)} - e^{-r(T-t_{n+1})}\right) \text{ if } E_t \geq E_{cap}.$$

A detailled discussion of the stability, consistency and convergence properties can be found in Hendricks (2013).

6.2 Monte Carlo Simulation of Clean Spread Options

The clean spread options in Section (2) will be computed via *Monte Carlo Simulations*. The underlying paths D, K and G will be simulated with the analytic solution of an Ornstein-Uhlenbeck process, while we will use a *Euler scheme* to approximate the evolution of cumulative emission. The EUA value is received by interpolating the discrete solution from our FDM scheme. As a tradeoff between accuracy and speed, a linear interpolation routine is used.

The option value is estimated by simulating N payoffs and paths, respectively, with maturity T

$$\hat{V}_{CDS} = e^{-rT} \frac{1}{N} \sum_{i=1}^{N} \left(S_T^i - \tfrac{1}{eff_1} \left(K_T^i + e_1 \, A_T^i \right) \right)^+ ,$$
$$\hat{V}_{CSS} = e^{-rT} \frac{1}{N} \sum_{i=1}^{N} \left(S_T^i - \tfrac{1}{eff_2} \left(G_T^i + e_2 \, A_T^i \right) \right)^+ ,$$

where the index i denotes the i-th simulated path and S is derived by solving equation (3.2).

7 Conclusion

In this paper we have computed clean spread options for the German electricity market. Based on a stochastic bid stack function, which connects demand and fuel with the electricity price, we have analysed EPEX Phelix bid data. The high supply from renewable energy and the regulations, concerning the forced usage of green energy, make the German energy market relatively unique in the world. The bid stack function could be extended to cope with the singularities of the German market. Furthermore, we have introduced the price of EUAs as an additional price driver. Based on the bid stack model, we implemented an emission rate, which connects fundamental factors and emission in a reasonable way.

The value of EUAs could be evaluated by solving a PDE with four spatial dimensions. It was solved numerically by an ADI scheme, based on a Crank-Nicolson approximation. The value of a power plant was computed with a real option approach in form of clean spread options. Our numerical simulations pointed out, that the spread significantly depends on the emission cap, the demand for electricity, the fuel prices and on the efficiency of a certain power plant. If the cap is determined carefully, thus leading to a working carbon market, clean plants are pushed forward in the merit order. The values of dirty and inefficient power plants drop to low levels and the incentive to invest in new technologies or switch to a cleaner fuel rises. In the high demand scenario, gas power plants could greatly benefit from higher electricity and EUA prices. In the third case study we have analysed the influence of changes in initial fuel prices. Although the fuel processes are mean reverting, we could see an influence on the value of power plants. Opposed to our intuitive expectation that gas power plants profit from declining gas prices, the simulations showed that this does not hold. Their value is much more influenced by the bids of their competitors (coal power plants).

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