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Nonlinear Model Order Reduction Based on Trajectory Piecewise Linear Approach: Comparing Different Linear Cores

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Abstract Refined models for MOS-devices and increasing complexity of circuit designs cause the need for Model Order Reduction (MOR) techniques that are capable of treating nonlinear problems. In time-domain simulation the Trajectory PieceWise Linear (TPWL) approach is promising as it is designed to use MOR methodologies for linear problems as the core of the reduction process. We compare different linear approaches with respect to their performance when used as kernel for TPWL.

1 Introduction

The tendency to analyze and design systems of ever increasing complexity is becoming more and more a dominating factor in progress of chip design. Along with this tendency, the complexity of the mathematical models increases both in structure and dimension. Complex models are more difficult to analyze, and due to this it is also harder to develop control algorithms. Therefore Model Order Reduction (MOR) is of utmost importance. For linear systems, quite a number of approaches are well-established and have proved to be very useful [1]. However, accurate models for MOS-devices introduce highly nonlinear equations. And, as the packing density in circuit design is growing, very large nonlinear systems arise. Hence, there is a growing request for reduced order modeling of nonlinear problems. In transient analysis the Trajectory PieceWise Linear (TPWL) approach [2, 3] is a promising technique as it makes use of linear MOR methods. A brief introduction to TPWL is given be-

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low. Analyzing the TPWL approach, we are interested in how different linear MOR techniques perform when used as a linear kernel, how robust the reduced models are and how they behave when combined to more complex systems.

2 MOR for linear problems

A continuous time-invariant (lumped) multi-input multi-output linear dynamical system is of the form:

$$\begin{cases} C\frac{dx(t)}{dt} = -Gx(t) + Bu(t), \\ y(t) = Lx(t) + Du(t), \quad x(0) = x_0, \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the inner state, $u(t) \in \mathbb{R}^m$ is the input, $y(t) \in \mathbb{R}^p$ is the output. The dimension n of the state vector is called the order of the system. C, G, B, L and D are the state space matrices. The dimension n of the system exhibits the order of elements contained in the circuit. As VLSI systems exhibit a large density of elements, n can easily reach a million.

Basically, MOR techniques aim to derive a system:

$$\begin{cases} \tilde{C}\frac{d\tilde{x}(t)}{dt} = -\tilde{G}\tilde{x}(t) + \tilde{B}u(t), \quad \tilde{x}(t) \in \mathbb{R}^q, \\ \tilde{y}(t) = \tilde{L}\tilde{x}(t) + \tilde{D}u(t), \quad \tilde{x}(0) = \tilde{x}_0, \ \tilde{y}(t) \in \mathbb{R}^p, \end{cases} \tag{2}$$

of order q with $q \ll n$ that can replace the original high-order system (1) in the sense, that the input-output behavior, described by the transfer function in the frequency domain, of both systems agrees. A common way is to identify a subspace of dimension $q \ll n$, that captures the dominant information of the dynamics and project (1) onto this subspace, spanned by some basis vectors $\{v_1, \ldots, v_q\}$.

The reduction can be carried out by means of different techniques. Approaches like PRIMA [4], SPRIM [5], and PMTBR [6] project the full problem (1) onto a subspace of dimension q. The first two rely on Krylov subspace methods. The latter one exploits the direct relation between the multipoint rational projection framework and the Truncated Balanced Realization (TBR). This approach can take advantage of some a-priori knowledge of the system properties, and is based on a statistical interpretation of the system Gramians. We give a brief review on these techniques and analyze their behavior when used as linear kernels in TPWL.

2.1 Krylov Projection Techniques and Poor Man's TBR

In recent years, MOR techniques based on Krylov subspaces have become the methods of choice for generating macromodels of large multi-port RLC circuits. Krylov subspace methods provide numerically robust algorithms for generating a basis of the reduced space, such that a certain number of moments of the transfer function of the original system is matched. Consequently, the transfer function of the reduced system approximates the original transfer functions around a specified frequency, or a collection of frequency points [7]. Owing to their robustness and low computational cost, Krylov subspace algorithms proved suitable for the reduction of large-scale systems, and gained considerable popularity, especially in electrical engineering. A number of Krylov-based MOR algorithms have been developed, including techniques based on the Lanczos method [8, 9] and the Arnoldi algorithm [4, 10]. The main drawbacks of these methods are, in general, lack of provable error bounds for the extracted reduced models, and no guarantees for preserving stability and passivity. Nevertheless, it has been demonstrated that if the original system has a specific structure, both stability and passivity can be preserved in the reduced system, by exploiting the fact that congruence transformations preserve the definiteness of a matrix. PRIMA [4] combines the moment matching approach with projection to arrive at a reduced system of type (2). Its main feature is that it produces provably passive reduced models.

However, PRIMA does not preserve the structure of the system matrices which is of an interest when trying to realize the reduced model. SPRIM [5], an adaption of this method, preserves block structures of the circuit matrices and generates provably passive and reciprocal macromodels of multiport RLC circuits. The SPRIM models match twice as many moments as the corresponding PRIMA models obtained with the same amount of computational work. Also SPRIM is less restrictive to matrices C and G in system (1), see [11].

Poor Man's TBR (PMTBR) [6] is a projection MOR technique that exploits the direct relation between the multipoint rational projection framework and the Truncated Balanced Realization (TBR). More details on PMTBR can be found in [6]. In the following simulation we assume that $C = I$ and $D = 0$ in (1).

2.2 Examples

We consider the RLC ladder networks, illustrated in Figure 1.

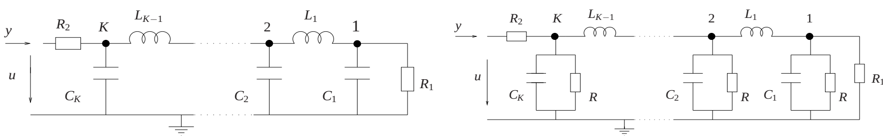

Fig. 1 Left: RLC circuit example 1; Right: RLC circuit example 2.

The state variable $x \in \mathbb{R}^{2K-1}$ consists of the voltages of the K nodes and the currents traversing the inductors $\{L_1, \ldots, L_{K-1}\}$. The voltage u and the current y represent input and output, respectively. Note that when the number of nodes is K the order of the system becomes $n = 2K - 1$.

Example 1. We choose an RLC ladder network shown in Figure 1 (left). We set all the capacitances and inductances to the same value 1 while $R_1 = \frac{1}{2}$ and $R_2 = \frac{1}{5}$, see [12]. We arrange 51 nodes which gives us the order 101 for the circuit.

Example 2. We use an RLC ladder network given in Figure 1 (right). We set all the capacitances and inductances to the same value 1 while $R_1 = \frac{1}{2}$, $R_2 = \frac{1}{5}$ and $R = 1$, we choose 51 nodes which results in order 101 for the circuit.

The main reason for choosing these two examples is the behavior of Hankel singular values, see [1]. The Hankel singular values for the first example do not show any significant decay while in the second example we observe a rapid decay in the values. The model is reduced by three linear techniques (PRIMA, SPRIM and PMTBR) from order 101 to order 34 for both examples. Figure 2 shows the absolute error between the transfer function of the full system and the transfer function of the reduced system.

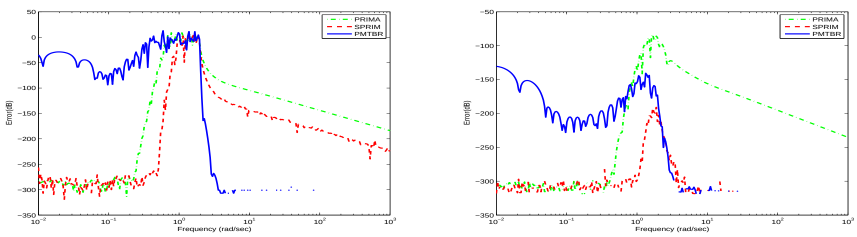

Fig. 2 Left: Error plot for the Example 1; Right: Error plot for the Example 2.

As we expected the SPRIM produces a better approximation than PRIMA since it matches twice as much moments. Although both methods have a good match around the expansion point 0, the error increases as we are far from the expansion point. As the Hankel singular values for the first example do not decay, the PMTBR cannot produce an accurate model for low frequency in that case. This shows that we can not stick to one method for reduction in general and the method should be chosen depending on the circuit behavior.

3 MOR for nonlinear problems

Large linear problems most frequently arise from modeling parasitic effects introduced by the layout, i.e., the wiring. As structure sizes decrease and packing densities increase the growing complexity of the nominal circuitry that is build up from transistors showing highly nonlinear behavior generates the need of MOR for nonlinear problems as well. In general an electric circuit can be described by a system of differential-algebraic equations (DAEs) of the form:

$$\frac{d}{dt}[q(x(t))] + j(x(t)) + Bu(t) = 0, \tag{3}$$

where $x(t) \in \mathbb{R}^n$ represents the unknown vector of circuit variables at time $t \in [t_0, t_e]$; the nonlinear functions $q, j : \mathbb{R}^n \to \mathbb{R}^n$ describe the contribution of reactive and nonreactive elements, respectively, and the matrix B distributes the input excitation $u : [t_0, t_e] \to \mathbb{R}^m$. Note that we concentrate on the state x only and omit the output stage y in our consideration.

MOR techniques developed for linear problems (1) cannot be applied directly to nonlinear models (3) as the transfer to a lower dimensional problem does not guarantee a reduction in the computational effort from evaluating the nonlinear model.

3.1 Trajectory Piecewise Linearization

The idea of TPWL [2, 3] is to represent the full nonlinear system (3) by a bunch of order reduced linear models that can reproduce the typical behavior of the system.

For this purpose a training input $\overline{u}(t)$ for $t \in [t_0, t_e]$ is chosen and a transient simulation is run in order to get a trajectory, i.e., a collection of points $\overline{x}_0, \ldots, \overline{x}_N$ approximating $x(t_i)$ at time-points $t_0 < t_1 < \cdots < t_N = t_e$, that reflect typical states of the system. On the trajectory, points $\{x_1^{\text{lin}}, \ldots, x_s^{\text{lin}}\} \subset \{\overline{x}_0, \ldots, \overline{x}_N\}$ are selected around which the nonlinear functions q and j are linearized. To the linear models, that are all of dimension n, any MOR for linear problems can be applied. This delivers local reduced subspaces $V_1, \ldots, V_s$ of possibly different dimensions $k_1, \ldots, k_s$. One common subspace V of dimension $k \ll n$ is constructed that describes the primary information of all local subspaces and on which all linear models are projected. Finally a weighting $w_i(Vz) \in [0, 1]$ for $i = 1, \ldots, s$ with $\sum_{i=1}^{s} w_i(Vz) = 1$ is introduced to decide which linear submodels are valid in a certain situation. The full system shall be replaced by the reduced one given by

$$\sum_{i=1}^{s} w_i(Vz) \left[V^T C_i V \frac{d}{dt} z + V^T G_i V z + V^T \left(j(x_i^{\text{lin}}) - G_i x_i^{\text{lin}} \right) \right] + V^T B u(t) = 0 \tag{4}$$

with $C_i = \left.\frac{\partial q}{\partial x}\right|_{x = x_i^{\text{lin}}}$ and $G_i = \left.\frac{\partial j}{\partial x}\right|_{x = x_i^{\text{lin}}}$

Besides the freedom in choosing which linear MOR technique to use there are also different strategies reported for determining the linearization points along the trajectory. In our considerations we stick to the strategy described in [3]. There at each time-point t_i both the full nonlinear system and the currently responsible reduced linear model are discretized with the same stepsize leading to two different approximations $\overline{x}_i$ and $\hat{x}_i = Vz_i$. Whenever the difference $\overline{x}_i - \hat{x}_i$ becomes too large, a new linearization point is arranged.

3.2 Example

We apply only PRIMA and PMTBR as a linear core for TPWL. In all simulation below the PMTBR is used unless stated otherwise. One of the partitions which is used inside the SPRIM algorithm is always of size 2 by 2 and the other part becomes larger as there is no inductor in the structure of the inverter chain. Therefore SPRIM is not reasonable to apply in this test case. The inverter chain constitutes a special class of circuit problems. Here a signal passes through the system, activating at each time-slot just a few elements and leaving the others latent. However, as the signal passes through, each element is active at some time and sleeping at some others. As in [13], the training of the inverter chain during the TPWL model extraction was done with a single piecewise linear input voltage at $\overline{u}(t)$ (see also Figure 3), defined by

$$\overline{u}(0) = 0,\ \overline{u}(5\text{ns}) = 0,\ \overline{u}(10\text{ns}) = 5,\ \overline{u}(15\text{ns}) = 5,\ \overline{u}(17\text{ns}) = 0.$$

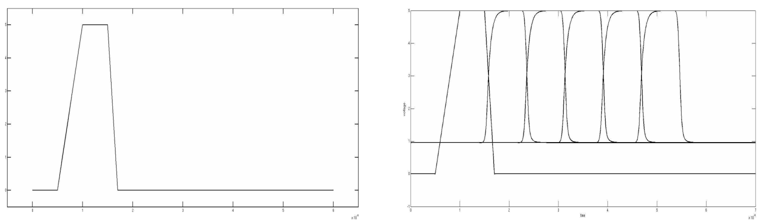

Fig. 3 Inverter chain: training input (left) and state response (right, all stages).

In Figure 4 we see the danger of defining distances to linearization points not in the full space but in the reduced space. Both plots are showing the signal at inverter 24. In Figure 4 in the right plot the second impulse is just not recognized where this seems to be no problem in the left plot. However, something else seems to be missing, even if we take the distance in the full space. In Figure 5 the voltage at inverters 68 and 92 is given. In both cases, the signal cannot be recovered correctly. In the latter one it is even not recognized at all. At the moment we cannot state reasons for that. Obviously this is not caused by the reduction but by the linearization or the weighting procedure as we get similar results when turning off the reduction step.

The impact of broadening the input signal u can be seen in Figure 6, which displays the voltage at inverters 18 and 68. The signals are far away from the expected behavior. However, there seems to be a trend towards the situation that was encountered during the training. And indeed in Figure 6 (right), at inverter 68 we find a time shifted version of the training signal instead of the wide input signal that has been applied now.

Finally, in Figure 7 the result of using the reduced model that arises from training input $\overline{u}$ of given pulse width with a slightly tighter input signal u is given for the inverters 6 and 68, respectively. In the former the characteristic is reflected quite well. However, in the latter the output signal seems to be just a time shifted version of the situation during the training. Having a closer look at how the inverter chain is

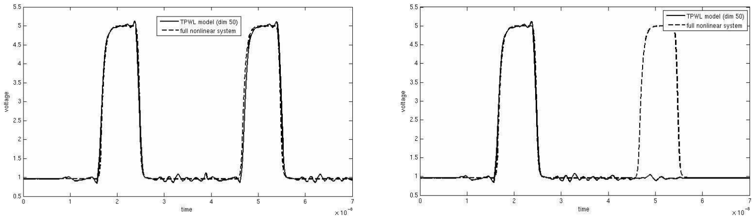

Fig. 4 Inverter chain: TPWL-resimulation, reduction to order 50, repeated pulse, inverter 24, Left: distance defined in full space; Right: distance defined in reduced space.

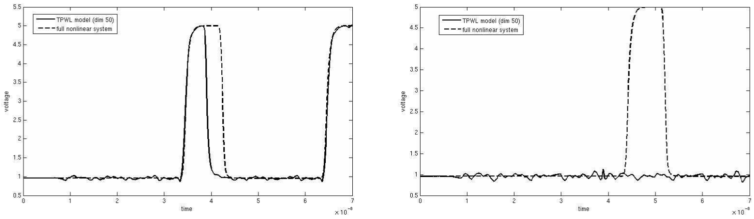

Fig. 5 Inverter chain: TPWL-resimulation, reduction to order 50, repeated pulse, distance in full space, Left: inverter 68; Right: inverter 92.

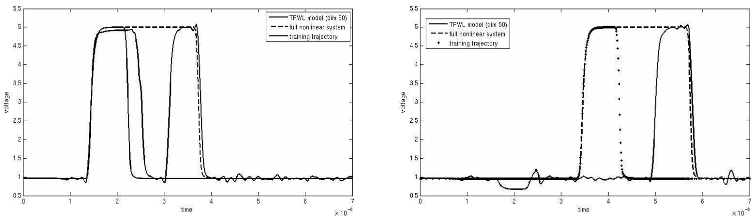

Fig. 6 Inverter chain: TPWL-resimulation, reduction to order 50, wider pulse, distance in full space, Left: inverter 18; Right: inverter 68.

modeled we see that the input voltage is applied at a floating node. This could give reasoning for the behavior encountered. However, also the backward and forward validity of the linear models could be the reasons.

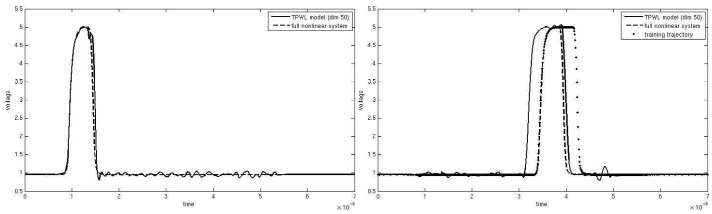

Fig. 7 Inverter chain: TPWL-resimulation, reduction to order 50, tighter impulse, distance in full space, Left: inverter 6; Right: inverter 68.

The error in Figure 8 is an overall error for all nodes. This total error shows that PMTBR yields better approximations than PRIMA. As changing from one linear method to the other the problems stay the same. Thus the reduction steps do not cause them.

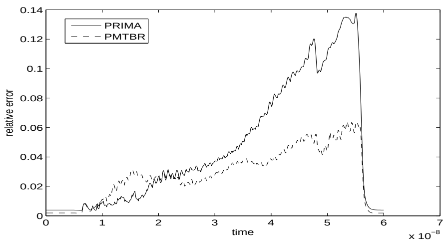

Fig. 8 Overall error for PRIMA and PMTBR used inside TPWL.

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