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Simulation of Pulsed Signals in MPDAE-Modelled SC-Circuits

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Summary. The simulation of circuits including signals with widely separated time scales can easily become very time-consuming. To avoid this, a multidimensional signal model was developed. The resulting system of network equations can be solved very efficiently by a method of characteristics. We investigate the applicability of this method to circuits including digital signal structures. Moreover, systems given in linear-implicit form are solved using the multidimensional approach.

1 Introduction

Signals with widely separated time scales often arise in radio frequency application. To describe such signals more efficiently, a multidimensional model has been developed, which transfers the circuit's differential-algebraic equations (DAE) to a multirate system of partial differential-algebraic equations (MPDAE). A specially tailored method of characteristics has already been sucessfully used to solve MPDAE-modelled network equations governed by semi-explicit DAEs including harmonic signals [4].

Now, we want to apply the method of characteristics to MPDAE-modelled switched capacitor (SC) circuits. In those circuits, transistors are driven by high frequency pulses, which are characterized by a digital signal structure.

In the first test example of a switched capacitor filter, the applicability of the method to the non-harmonic, strongly nonlinear signals is investigated. The second circuit of the Miller integrator serves to simulate network equations, which are given in linear-implicit form.

2 Switched capacitor filter

The first test example is the switched capacitor filter depicted in figure 1. A sinusoidal input signal charges the first capacitor driven by the pulse p_a and

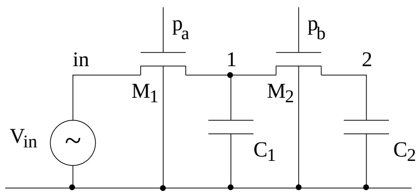

Fig. 1. Switched capacitor filter

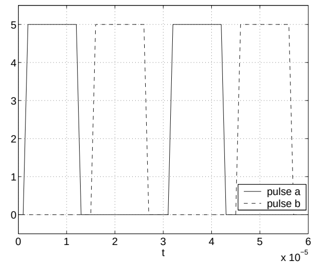

Fig. 2. Pulses p_a and p_b

this charge is transmitted to the second capacitor driven by the pulse p_b. The transistors work as switches and the pulses have to be complementary to each other as shown in figure 2. The equations for the two nodes are in ODE form given by

$$\begin{aligned} -I_{DS}(u_{p_a}, u_{in}, u_1, 0) + I_{DS}(u_{p_b}, u_1, u_2, 0) + C_1 \dot{u}_1 & \\ + CGSO \cdot W \cdot \frac{d(u_1 - u_{p_a})}{dt} + CGDO \cdot W \cdot \frac{d(u_1 - u_{p_b})}{dt} &= 0 \qquad (1) \\ -I_{DS}(u_{p_b}, u_1, u_2, 0) + C_2 \dot{u}_2 + CGSO \cdot W \cdot \frac{d(u_2 - u_{p_b})}{dt} &= 0 \qquad (2) \end{aligned}$$

with overlap capacitances $CGSO$, $CGDO$ and transistor width W. For the drain to source current $I_{DS}(u_{gate}, u_{drain}, u_{source}, u_{bulk})$ of the MOS-transistors M_1 and M_2, a level-1 model by Stichman-Hodges is used [1].

The pulses work at the fast time scale $T_2 = 3 \cdot 10^{-5}$ s, whereas the sinusoidal input V_{in} oscillates with $T_1 = 10^{-3}$ s. To describe these widely separated time scales more efficiently, a multidimensional signal model is applied. A detailed description of this modelling approach can be found in [2].

3 Multidimensional approach

To decouple the different time scales of the switched capacitor circuit, a corresponding variable is assigned to each of them. For two different time scales this approach generalizes a two-tone signal $s(t)$ to a so-called multivariate funtion (MVF) $\hat{s}(t_1, t_2)$, for example

$$s(t) = \sin\left(\frac{2\pi}{T_1} t\right) \sin^2\left(\frac{\pi}{T_2} t\right) \quad \rightsquigarrow \quad \hat{s}(t_1, t_2) = \sin\left(\frac{2\pi}{T_1} t_1\right) \sin^2\left(\frac{\pi}{T_2} t_2\right).$$

The original signal can always be reconstructed by $s(t) = \hat{s}(t, t)$.

Applying this multidimensional signal model to the SC-filter circuit transfers the network-ODE (1)+(2) to a multirate partial differential equation (MPDE):

$$
\begin{aligned}
&(C_1 + CGSO \cdot W + CGDO \cdot W)\left(\frac{\partial u_1(t_1,t_2)}{\partial t_1} + \frac{\partial u_1(t_1,t_2)}{\partial t_2}\right)\\
&\quad = I_{DS}(u_{p_a}(t_2), u_{in}(t_1), u_1(t_1,t_2), 0) - I_{DS}(u_{p_b}(t_2), u_1(t_1,t_2), u_2(t_1,t_2), 0)\\
&\qquad + CGSO \cdot W \cdot \frac{du_{p_a}(t_2)}{dt_2} + CGDO \cdot W \cdot \frac{du_{p_b}(t_2)}{dt_2}
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
&(C_2 + CGSO \cdot W)\left(\frac{\partial u_2(t_1,t_2)}{\partial t_1} + \frac{\partial u_2(t_1,t_2)}{\partial t_2}\right)\\
&\quad = I_{DS}(u_{p_b}(t_2), u_1(t_1,t_2), u_2(t_1,t_2), 0) + CGSO \cdot W \cdot \frac{du_{p_b}(t_2)}{dt_2}\ .
\end{aligned}
\tag{4}
$$

As the PDE is of hyperbolic type, we are able to apply the method of characteristics described in [4]. The ODEs arising in the characteristic system of the MPDE are solved via discretization along the characteristic curves, which are straight lines in the direction of the diagonal. Boundary conditions are given by the periodicity of the MVFs. The simulation results for node 2, which coincide with solutions generated by MATLAB-routines, are shown in figure 3.

Thus, the application of the method of characteristics to network equations including digital signal structures works successfully. In the following, we investigate a system given in a linear-implicit form.

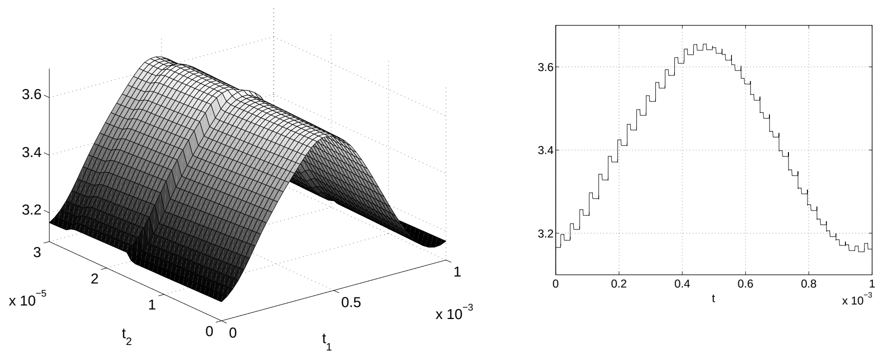

Fig. 3. MPDE-solution (left), reconstructed ODE-solution (right)

4 Miller integrator

The Miller integrator in figure 4 produces the negative integral of the input signal at node 3. The sinusoidal input with period $T_1 = 10^{-5}\,$s is sampled periodically with $T_2 = 25 \cdot 10^{-9}\,$s. Pulses p_a and p_b have a similar behaviour as above (see figure 2).

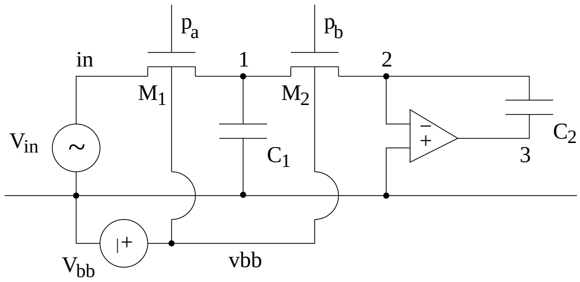

Fig. 4. Miller integrator

How the index of the network equations may depend on the value of technical circuit parameters is investigated in [3]. In our example, the network equations are an index-1 DAE-system. Again, bivariate functions are introduced for all state variables and sources, which leads to multirate partial differential-algebraic equations (MPDAE) given in a linear-implicit form:

$$\begin{aligned} C_1 \cdot \left(\frac{\partial u_1(t_1,t_2)}{\partial t_1} + \frac{\partial u_1(t_1,t_2)}{\partial t_2} \right) &= I_{DS}(u_{p_a}(t_2), u_{in}(t_1), u_1(t_1,t_2), vbb) \\ &- I_{DS}(u_{p_b}(t_2), u_1(t_1,t_2), u_2(t_1,t_2), vbb) \end{aligned} \tag{5}$$

$$\begin{aligned} C_2 \cdot &\left(\frac{\partial\,[u_2(t_1,t_2) - u_3(t_1,t_2)]}{\partial t_1} + \frac{\partial\,[u_2(t_1,t_2) - u_3(t_1,t_2)]}{\partial t_2} \right) \\ &= I_{DS}(u_{p_b}(t_2), u_1(t_1,t_2), u_2(t_1,t_2), vbb) \end{aligned} \tag{6}$$

$$0 = u_3(t_1,t_2) + 1000 \cdot u_2(t_1,t_2) \tag{7}$$

with a negative substrate bias voltage vbb.

Again, the method of characteristics described in the previous section was used to solve the system. Also for this example, the one-dimensional solution reconstructed from the MPDAE-solution coincides well with a corresponding Matlab-solution of the original network equations. Figure 5 shows the simulation results for node 1. Thus, equations given in linear-implicit form can also be solved via the multidimensional approach.

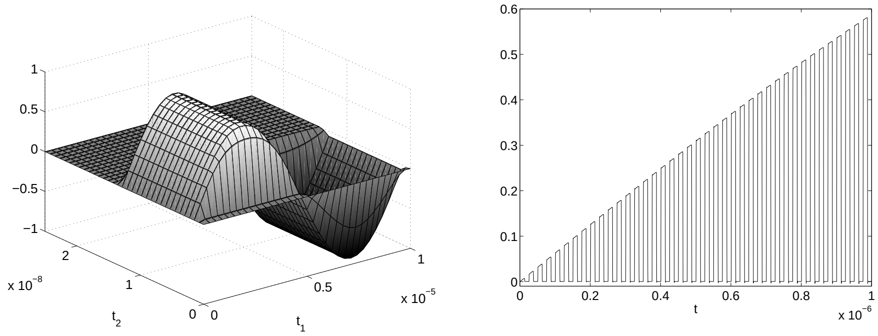

Fig. 5. MPDAE-solution (left), zoom into DAE-solution (right)

5 Conclusions

The approach via characteristic systems was successfully applied to MPDAE-modelled pulsed signals in switched capacitor circuits. Not only harmonic but also digital-like signals can be simulated using the described method of characteristics. In addition, network equations given in linear-implicit form can be solved as well as explicit ones. In any case, the efficiency of the multidimensional approach and of the specially tailored method can be exploited.

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