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Summary. In radio frequency (RF) application, electric circuits often exhibit multitone signals, where time scales differ by several orders of magnitude. Thus circuit simulation by means of transient analysis becomes inefficient. A multivariate model yields an alternative strategy considering amplitude as well as frequency modulation. Consequently, a warped multirate partial differential algebraic equation (MPDAE) has to be solved using periodic boundary conditions. Thereby, the determination of a local frequency function is crucial for the efficiency of the model. For this purpose, two special choices of continuous phase conditions are applied as additional boundary conditions. Numerical simulations show that these continuous phase conditions identify local frequency functions, which are physically reasonable.

Key words: multirate partial differential algebraic equation, phase condition, circuit simulation, frequency modulation, radio frequency.

1 Introduction

Numerical simulation of electric circuits rests upon a network approach, which yields systems of differential algebraic equations (DAEs), see [GF99]. In RF application, generated signals often exhibit widely separated time scales. For example, a slow oscillation may vary the amplitude of a carrier wave. Therefore a transient integration of the DAE system becomes costly, since the fastest rate restricts the step size.

A signal model using multivariate functions (MVFs) decouples the time scales and thus provides an alternative strategy. Consequently, Brachtendorf et al. [BWLB96] introduced a multirate partial differential algebraic equation (MPDAE), which allows the simulation of amplitude modulated signals in forced oscillators. If the circuit also includes autonomous time scales, frequency modulation may result, too. Narayan and Roychowdhury [NR03] generalised the model into a warped MPDAE for this case. Accordingly, a time-dependent local frequency function arises, which influences essentially the signal representation. However, an appropriate choice of the local frequency is unknown at the beginning.

We use continuous phase conditions to determine the local frequency function by the behaviour of corresponding MVFs. Thereby, the idea is to control the phase in slice planes of the MVF. This strategy yields additional boundary conditions for the warped MPDAE system in time domain. We apply this technique to a forced Van der Pol oscillator.

2 Multivariate Signal Model

To illustrate the multidimensional signal model, we consider a simple multitone oscillation

$$x(t) = \left[1 + \alpha \sin\left(\tfrac{2\pi}{T_1} t\right)\right] \sin\left(\tfrac{2\pi}{T_2} t + \beta \sin\left(\tfrac{2\pi}{T_1} t\right)\right) \qquad (1)$$

for parameters $0 < \alpha < 1$, $\beta > 0$. If $T_1 \gg T_2$ holds, then a high-frequency oscillation arises, where amplitude as well as frequency is modulated by a slow oscillation. Hence we need many time steps to resolve this signal accurately. Alternatively, an own variable is introduced for each separate time scale, which yields directly the biperiodic function

$$\hat{x}_1(t_1, t_2) = \left[1 + \alpha \sin\left(\tfrac{2\pi}{T_1} t_1\right)\right] \sin\left(2\pi t_2 + \beta \sin\left(\tfrac{2\pi}{T_1} t_1\right)\right), \qquad (2)$$

where the second period is transformed to 1. We can completely reconstruct the original signal via $x(t) = \hat{x}_1(t, t/T_2)$. This representation (2) is called a MVF of the multitone signal (1). Unfortunately, the MVF (2) exhibits many oscillations in the rectangle $[0, T_1[\times [0, 1[$ for large parameters β. Thus we include only the amplitude modulation part in a MVF, i.e.

$$\hat{x}_2(t_1, t_2) = \left[1 + \alpha \sin\left(\tfrac{2\pi}{T_1} t_1\right)\right] \sin\left(2\pi t_2\right). \qquad (3)$$

Now the function features a simple behaviour in $[0, T_1[\times [0, 1[$. Therefore we can represent this MVF with sufficient accuracy using relatively few grid points. The frequency modulation part is modelled by a separate function

$$\Psi(t) = \frac{t}{T_2} + \frac{\beta}{2\pi} \sin\left(\tfrac{2\pi}{T_1} t\right). \qquad (4)$$

Now we are able to reconstruct the signal (1) applying $x(t) = \hat{x}_2(t, \Psi(t))$. The derivative $\nu := \Psi'$, which is a T_1-periodic time-dependent function, can be seen as a local frequency of the signal. Thus we obtain an efficient representation by means of this model.

Using the inappropriate MVF (2), the reconstruction formula indicates a local frequency $\nu \equiv 1/T_2$. It follows that the choice of a local frequency function is not unique and critical for the efficiency of the MVF model.

3 Warped MPDAE System

In general, an electric circuit is modelled by a DAE system of the form

$$\frac{\mathrm{d}\mathbf{q}(\mathbf{x})}{\mathrm{d}t} = \mathbf{f}(\mathbf{x}) + \mathbf{b}(t) \qquad \big(\mathbf{x}(t), \mathbf{b}(t), \mathbf{q}(\mathbf{x}), \mathbf{f}(\mathbf{x}) \in \mathbb{R}^k\big)\,, \tag{5}$$

where $\mathbf{x}$ denotes unknown voltages and currents. The input signals $\mathbf{b}$ shall be T_1-periodic. We assume that $\mathbf{x}$ is a multitone signal of the discussed type. Applying the multivariate model, the DAE changes into the MPDAE

$$\frac{\partial\mathbf{q}(\hat{\mathbf{x}})}{\partial t_1} + \nu(t_1)\frac{\partial\mathbf{q}(\hat{\mathbf{x}})}{\partial t_2} = \mathbf{f}(\hat{\mathbf{x}}) + \mathbf{b}(t_1) \qquad \big(\hat{\mathbf{x}}(t_1,t_2) \in \mathbb{R}^k,\ \nu(t_1) \in \mathbb{R}\big) \tag{6}$$

with the MVF $\hat{\mathbf{x}}$ of $\mathbf{x}$. It follows that a $(T_1, 1)$-periodic MPDAE solution yields multitone DAE solution via $\mathbf{x}(t) = \hat{\mathbf{x}}(t, \int_0^t \nu(\tau)d\tau)$. Thereby, the T_1-periodic local frequency ν is a priori unknown and thus the system (6) is underdetermined. Houben [Hou04] proposed minimum conditions, which reduce oscillatory behaviour in MVFs, to fix this function.

Alternatively, we try to control the phase in each slice plane of the MVF for constant t_1. A unifying effect shall produce simple MVF representations. Since the local frequency is a scalar function, we consider just a single component of the MVF $\hat{\mathbf{x}} = (\hat{x}^1, \dots, \hat{x}^k)^T$, for example the first one. Now feasible choices for continuous phase conditions are

$$\hat{x}^1(t_1, 0) = \eta \quad (\eta \in \mathbb{R}) \quad \text{for all } t_1 \tag{7}$$

or

$$\left.\frac{\partial \hat{x}^1}{\partial t_2}\right|_{t_2=0} = 0 \quad \text{for all } t_1. \tag{8}$$

Consequently, we add either (7) or (8) to the biperiodic boundary conditions in a time domain method. Thus the resulting technique is cheaper in comparison to a minimisation procedure. The existence of MVFs satisfying one of the phase conditions can be motivated by transformations of MPDAE solutions.

4 Numerical Simulation

As benchmark, we consider a forced Van der Pol oscillator of the form

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -10(x^2-1)y + (2\pi z)^2 x \\ 0 &= z - \left[1 + \tfrac{1}{2}\sin\left(2\pi 10^{-3} t\right)\right], \end{aligned} \tag{9}$$

which represents a DAE system of index 1. A multitone solution arises and we employ the warped MPDAE model. Numerical solutions are obtained by a time domain technique, which is based on characteristic curves, see [Pul04].

Let ν_a and ν_b be the local frequencies, which are caused by the phase conditions (7) and (8), respectively. Fig. 1 illustrates these functions, which are nearly the same ($|\nu_a - \nu_b| < 10^{-3}$). Since the frequencies respond to the input, they are physically reasonable. The corresponding MVFs $\hat{x}$ and $\hat{y}$ are shown in Fig. 2. The solutions belonging to the two phase conditions differ mainly by a translation in t_2-direction, which reflects that (6) is autonomous in the variable t_2. Although $\hat{x}$ exhibits nearly constant amplitude, $\hat{y}$ includes amplitude modulation. Finally, Fig. 3 displays the reconstructed DAE solution x together with a reference solution of (9). We observe a phase shift in later cycles. Nevertheless, the other signal properties coincide at any time.

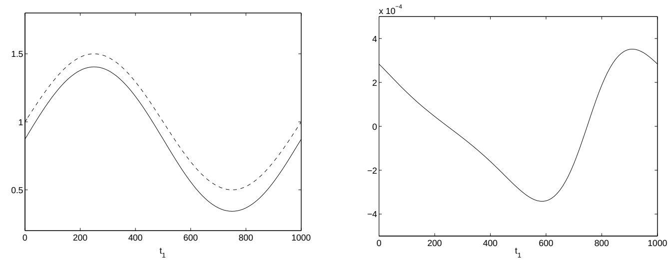

Fig. 1. Local frequency ν_a (solid line) together with input signal (dashed line) (left) and difference of local frequencies $\nu_a - \nu_b$ (right).

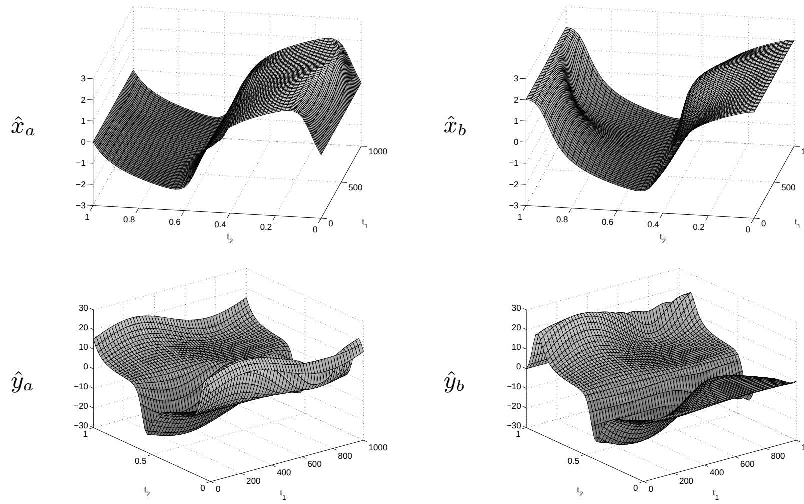

Fig. 2. MPDAE solutions using phase condition (7) (left) and (8) (right).

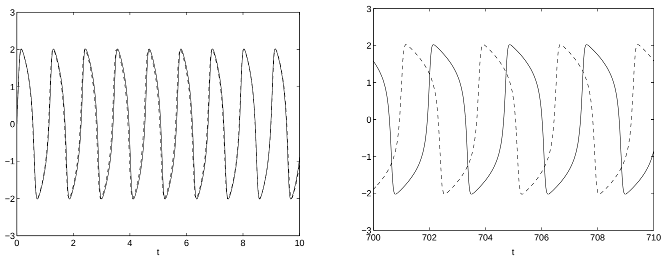

Fig. 3. DAE solution x integrated by trapezoidal rule (solid line) and interpolated by MPDAE solution (dashed line) in time intervals $[0, 10]$ (left) and $[700, 710]$ (right).

5 Conclusions

A multivariate model for analysing oscillators, which produce amplitude as well as frequency modulated signals, has been presented. The arising MPDAE system demands the identification of an appropriate local frequency function. Numerical simulations demonstrate that continuous phase conditions are able to determine physically reasonable local frequencies. Thus corresponding MVFs exhibit a simple structure and the model becomes efficient. Underlying existence theorems using the phase conditions still have to be researched.

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