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Jump-diffusion models with two stochastic factors for pricing swing options in electricity markets with partial-integro differential equations [1]

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Abstract

In this paper we consider the valuation of swing options with the possibility of incorporating spikes in the underlying electricity price. This kind of contracts are modelled as path dependent options with multiple exercise rights. From the mathematical point of view the valuation of these products is posed as a sequence of free boundary problems where two consecutive exercise rights are separated by a time period. Due to the presence of jumps, the complementarity problems are associated with a partial-integro differential operator. In order to solve the pricing problem, we propose appropriate numerical methods based on a Crank-Nicolson semi-Lagrangian method for the time discretization of the differential part of the operator, jointly with the explicit treatment of the integral term by using the Adams-Bashforth scheme and combined with biquadratic Lagrange finite elements for space discretization. In addition, we use an augmented Lagrangian active set method to cope with the early exercise feature. Moreover, we employ appropriate artificial boundary conditions to treat the unbounded domain numerically. Finally, we present some numerical results in order to illustrate the proper behaviour of the numerical schemes.

Keywords: Swing options; electricity price; jump-diffusion models; Augmented Lagrangian Active Set(ALAS) formulation; semi-Lagrangian method; biquadratic Lagrange finite elements; artificial boundary conditions.

1 Introduction

Swing contracts are a kind of path-dependent option that allows the holder to exercise a right multiple times over a period, with the constraint that the two consecutive exercise dates must be separated by a refracting period. That is, two rights can not be exercised at the same time.

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One example of this right could be to receive the payoff of a call option. Nevertheless, there are other possibilities, such as the consideration of different payoff functions depending on the spot price, such as calls and puts, or calls with different strikes. In this work we focus on the valuation of swing options where the underlying is the electricity price. In order to model some properties of the electricity prices observed in the markets, such as spikes, it is necessary to adopt jump-diffusion models. According to [16], these sudden changes in the prices appear when the maximum supply is approached by the current demand. Concerning risk management and pricing, it is very important to treat properly these features of the electricity prices. The consideration of jumps in the electricity prices is the main innovative point of the present paper. In the literature, there are several examples of the valuation of financial derivatives when the underlying assets follow a jump-diffusion process (see [29], for example). Among them, in this work we assume that the spike sizes follow an exponential distribution [16], or the spikes are governed by Merton [25] and Kou [22] jump-diffusion models. The consideration of jump-diffusion models leads to a partial-integro differential equation (PIDE) problem.

In the absence of jumps, different numerical techniques have been employed to deal with the one-factor swing option valuation problem. Binomial trees are considered in [18], while Monte Carlo simulation techniques are employed in [24] and [32]. As indicated in [6], the numerical resolution of the PDE approach with finite elements or finite differences has been carried out in [10], [33] and [34]. In the literature, also Fourier-based methods have been employed to treat this valuation problem in [35].

Taking into account the possibility of jumps in the electricity price, the two-factor swing option valuation problem is solved by using binomial trees in [16]. When jump-diffusion processes are considered, a partial-integro differential equation arises. Thus, in [20] the one-factor PIDE is solved by using a finite-difference scheme combined with a dynamic programming technique, while in [26] an implicit-explicit finite difference scheme is proposed. The finite difference method jointly with the approximation of the integral term by means of a recursion formula is also implemented in [4] for the one factor problem under Kou jump-diffusion model.

In the present paper, we propose the numerical solution of the PIDE model that governs the valuation of swing contracts depending on two stochastic factor with the possibility of incorporating spikes. Concerning the numerical methods for solving PIDE problems arising in finance, in [12] the authors propose a semi-Lagrangian method for pricing American Asian options assuming jump-diffusion models for the underlying asset while in [8] an implicit finite difference method to obtain the value of options on two assets under jump-diffusion processes is considered. Moreover, in order to avoid the solution of a linear system with a dense matrix, they combine a fixed point iteration with a FFT technique. In [1], the authors solve the PIDE to obtain the value of European vanilla options under Merton and Kou jump-diffusion models for the underlying.

In the presence of jumps, when the integral term is treated implicitly, the numerical schemes employed to solve the PIDE lead to a dense matrix, being necessary to employ appropriate methods to solve the system as the one proposed in [30]. In contrast when treating explicitly the integral term either by using a time discretization scheme which involves the solution of the previous time step [9] or of the two previous time steps [31] the same matrix as in the absence of jumps is maintained. In our previous work [5] we have chosen the first approach, whereas in this work we implement the second one, treating explicitly the integral term by taking into account the solution in the two previous time steps.

In order to solve the complementarity problems that arise in the valuation of products of American type, there exists different methods proposed in the literature. For example, in [11] a penalty method is proposed and in [17] an operator splitting method is presented.

In this paper we propose a Lagrange-Galerkin method for time and space discretization [2, 3], combined with an Augmented Lagrangian Active Set (ALAS) algorithm [19], which is more efficient than the classical PSOR, or alternative duality or penalization methods (see [6], for more detail). Moreover, we implement the explicit treatment of the integral term proposed in [31] (i.e. the Adams-Bashforth scheme).

This paper is organized as follows. In Section 2, we describe the stochastic model for the evolution of the electricity price under jump diffusion-processes we consider, and we state the PIDE problem that governs the valuation of swing options on this underlying. In Section 3, we formulate the swing option pricing problem with jumps in a bounded domain after a localization procedure. Next, we introduce the discretization in time of the problem, using a Crank-Nicolson characteristic scheme for the differential part of the operator jointly with the explicit treatment of the integral term, and we state the variational formulation of the discretized problem in order to apply finite elements. Finally, in Section 4, we present some numerical results and in Section 5 we finish with some conclusions.

2 The mathematical model under jump-diffusion processes

2.1 The electricity spot price model incorporating spikes

In our previous work [6], we considered a model without jumps to describe the dynamics of the electricity price. Nevertheless, in this paper we take into account the possibility of spikes in the prices, which is more realistic to represent the behaviour of the electricity prices in the markets. Thus, in this section we introduce the model to describe the stochastic evolution of the electricity spot price in the presence of jumps. More precisely, under the risk neutral probability measure, the spot electricity price, S_t, is assumed to be the continuous time process

$$S_t = \exp(f(t) + \bar{X}_t + Y_t), \tag{1}$$

where f is a deterministic periodic function that represents the seasonality and accounts for regular changes in the prices evolution, $\bar{X}_t$ denotes the Ornstein-Uhlenbeck (OU) process with zero mean-reversion level and mean-reversion speed, $\alpha > 0$. Thus, the following stochastic differential equation (SDE) is satisfied by the process $\bar{X}_t$:

$$d\bar{X}_t \;=\; -\alpha \bar{X}_t dt \;+\; \sigma dW_t \,, \tag{2}$$

where σ denotes the volatility of the process and W_t represents a standard Brownian motion. In order to incorporate spikes, for the third component, Y_t, we consider the following SDE:

$$dY_t \;=\; -(\beta + \lambda\kappa) Y_t dt + J_t dZ_t \,, \tag{3}$$

where β is the mean-reversion speed, J_t denotes the jump size distribution, Z_t is a compensated Poisson process of intensity λ and $\kappa = \mathbb{E}[\exp(J_t) - 1]$. Note that the mean-reversion rate of the jump process, Y_t, can be different to the one of the process $\bar{X}_t$ and even higher, thus allowing us to model spikes mainly in markets with low mean-reversion speed. For convenience, we write the seasonal function f as a time-dependent mean reversion level of the process $\bar{X}_t$ and we introduce $X_t = \bar{X}_t + f(t)$. For convenience, we consider the following processes

$$M_t = \exp(X_t), \quad N_t = \exp(Y_t),$$

so that $S_t = M_t N_t$ and the following SDEs are satisfied

$$\begin{aligned} dM_t &= \alpha\left(\mu(t) - \ln(M_t)\right) M_t dt + \sigma M_t dW_t\,, \\ dN_t &= -(\beta + \lambda\kappa)\ln(N_t) N_t dt + (\exp(J_t) - 1) N_t dZ_t\,, \end{aligned} \tag{4}$$

with

$$\mu(t) = f(t) + \frac{1}{\alpha}\left(\frac{\sigma^2}{2} + f'(t)\right).$$

In the absence of jumps, there exist other alternative models, either with one or two stochastic factors, to describe the spot electricity price evolution as the ones presented in [23].

2.2 The PIDE formulation

In the case without jumps, by using a dynamic hedging technique in [6], a partial differential equation (PDE) model for pricing any asset depending on the stochastic factors M_t and N_t (the dynamics of which are described without jumps in [6]) is posed. In the here considered jump-diffusion models, if we denote by $V_t = V(t, M_t, N_t)$ the price of any asset whose value is a function of time t and the stochastic factors M_t and N_t (the dynamics of which are described by equations (4)) then standard techniques based on Ito formulas for jump-diffusion process prove that the function V satisfies the following PIDE (see [9], for example):

$$\begin{aligned} &\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 M^2 \frac{\partial^2 V}{\partial M^2} + \alpha\left(\mu(t) - \ln(M)\right) M \frac{\partial V}{\partial M} - \beta \ln(N) N \frac{\partial V}{\partial N} - rV \\ &+ \int_{-\infty}^{\infty} \lambda \left[V(t, M, N\exp(z)) - V(t, M, N) - N(\exp(z) - 1)\frac{\partial V(t, M, N)}{\partial N}\right] \nu(z) dz = 0. \end{aligned} \tag{5}$$

Moreover, in order to complete the model, we must also specify the distribution of the jump sizes. For this purpose, we will consider either the Merton model [25] or the Kou model [22]. More precisely, under Merton model J follows the normal distribution ($N(\mu_j, \gamma_j^2)$), with the density function

$$\nu(z) = \nu_m(z) = \frac{1}{\gamma_j\sqrt{2\pi}} \exp\left(-\frac{(z - \mu_j)^2}{2\gamma_j^2}\right), \tag{6}$$

where μ_j is the mean jump size and γ_j is the standard deviation of the jump size, whereas under Kou model J corresponds to a distribution with double-exponential density function

$$\nu(z) = \nu_k(z) = \begin{cases} q_k \alpha_2 \exp(\alpha_2 z), & z < 0 \\ p_k \alpha_1 \exp(-\alpha_1 z), & z \geq 0, \end{cases} \tag{7}$$

where p_k, q_k, α_1 and α_2 are positive constants such that $p_k + q_k = 1$ and $\alpha_1 > 1$. Note that, p_k and q_k represent the probabilities of upward and downward jumps, respectively.

Since $\nu(z)$ is the probability density function of the jump amplitude J, then

$$\int_{-\infty}^{\infty} \nu(z)dz = 1.$$

Moreover, we can compute the expectations for the Merton and Kou models

$$\mathbb{E}_m[\exp(J)] = \int_{-\infty}^{\infty} \exp(z)\nu_m(z)dz = e^{\mu_j+\gamma_j^2/2},$$

$$\mathbb{E}_k[\exp(J)] = \int_{-\infty}^{\infty} \exp(z)\nu_k(z)dz = \frac{p_k\alpha_1}{\alpha_1 - 1} + \frac{q_k\alpha_2}{\alpha_2 + 1}.$$

Therefore, the PIDE (5) can be written in the equivalent form

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 M^2 \frac{\partial^2 V}{\partial M^2} + \alpha\left(\mu(t) - \ln(M)\right) M \frac{\partial V}{\partial M} - (\beta \ln(N) + \lambda\kappa) N \frac{\partial V}{\partial N} - (r+\lambda)V$$
$$+\lambda \int_{-\infty}^{\infty} V(t, M, N\exp(z))\nu(z)dz = 0. \tag{8}$$

where $\kappa = e^{\mu_j+\gamma_j^2/2} - 1$ or $\kappa = \frac{p_k\alpha_1}{\alpha_1-1} + \frac{q_k\alpha_2}{\alpha_2+1} - 1$ for the Merton or Kou models, respectively.

Note that with respect to the PDE model in [6], there is an additional integral term in Equation (8) due to the presence of jumps. This term makes the PIDE more difficult to solve than the corresponding PDE. In Section 3.4 we show how to discretize this integral term in order to find a numerical solution of the PIDE problem.

For the particular case of electricity markets in which the payoff $\phi(T, S)$ is a function depending on the electricity price, S, at maturity T, Equation (8) is supplied with the final condition

$$V(T, M, N) = \phi(T, MN), \tag{9}$$

where the function ϕ denotes the payoff of the contract.

2.3 The swing option pricing problem under jump diffusion models

From a mathematical point of view, swing options in electricity markets can be modelled as financial products with multiple exercises of American type. Moreover, two consecutive exercise dates are separated by a constant refracting period $\delta > 0$. As it is mentioned in [7], the consideration of this refracting period avoids the exercise of all the rights at once, which would be optimal in the absence of this separation time. That is, without the refracting period δ, the swing option pricing problem could be reduced to the valuation of multiple American options.

Let us consider $p \in \mathbb{N}$ exercise rights. If we denote by $\mathcal{T}_{t,T}$ the set of all stopping times with values in $[0, T]$ and by $\mathcal{T}_{t,\infty}$ the set of all stopping times with values greater or equal than t, then we can define the set of admissible stopping time vectors in the following way (see [7] or [34], for example)

$$\mathcal{T}_t^{(p)} = \{\tau^{(p)} = (\tau_1, ..., \tau_p) | \tau_i \in \mathcal{T}_{t,\infty} \text{ with } \tau_1 \leq T \text{ a.s. and } \tau_{i+1} - \tau_i \geq \delta \text{ for } i = 1, ..., p-1\}. \quad (10)$$

Note that at least one exercise right of the swing option with maturity T is exercised, although it is not necessary to exercise all the rights. The investor could let an exercise right expire to get a benefit from better future prices. Thus, not all stopping times of a stopping time vector have their values in the interval $[0, T]$.

In [34] the risk free price of a swing option depending on one underlying factor without jumps is written as a multiple stopping time problem and it is proved in [7] that it can be translated into a sequence of single stopping time problems. Analogously, in the presence of jumps and when the electricity price depends on two stochastic factors under a risk neutral probability measure Q, the price of a swing option with $p \in \mathbb{N}$ exercise rights is given by

$$V^{(p)}(t, M_t, N_t) = sup_{\tau \in \mathcal{T}_{t,T}} \mathbb{E}^Q \left[e^{-r(\tau - t)} \Phi^{(p)}(\tau, S_\tau) \right], \quad p \geq 1, \quad (11)$$

with $S_t = M_t N_t$ and

$$\Phi^{(p)}(t, S_t) = \begin{cases} \phi(t, S_t) + \mathbb{E}^Q \left[e^{-r\delta} V^{(p-1)}(t + \delta, M_{t+\delta}, N_{t+\delta}) \right] & \text{if } t \leq T - \delta \\ \phi(t, S_t) & \text{if } t > T - \delta. \end{cases}$$

Moreover, we start from:

$$V^{(0)}(t, M_t, N_t) = 0. \quad (12)$$

2.3.1 The free boundary problem under jump-diffusion models

In the presence of spikes in the electricity price, after making the time reversal change of variable, $\tau = T - t$, we introduce the function $u^{(p)}(\tau, M, N) = V^{(p)}(T - \tau, M, N)$ so that it solves the following complementarity problem:

$$\begin{aligned} \mathcal{L}[u^{(p)}] \leq 0 \quad & \text{in } (0, T) \times \mathbb{R}_+^2, \\ u^{(p)} \geq \Psi^{(p)} \quad & \text{in } (0, T) \times \mathbb{R}_+^2, \\ (\mathcal{L}[u^{(p)}])(u^{(p)} - \Psi^{(p)}) = 0 \quad & \text{in } (0, T) \times \mathbb{R}_+^2, \\ u^{(p)}(0, .) = \Psi^{(p)}(0, .) \quad & \text{in } \mathbb{R}_+^2, \end{aligned} \quad (13)$$

where the operator $\mathcal{L}$ is defined by

$$\begin{aligned} \mathcal{L}[F] = -\frac{\partial F}{\partial \tau} + \frac{1}{2}\sigma^2 M^2 \frac{\partial^2 F}{\partial M^2} + \alpha \left(\mu(T - \tau) - \ln(M) \right) M \frac{\partial F}{\partial M} - (\beta \ln(N) + \lambda\kappa) N \frac{\partial F}{\partial N} \\ -(r + \lambda) F + \lambda \int_{-\infty}^{\infty} F(\tau, M, N \exp(z)) \nu(z) dz \end{aligned} \quad (14)$$

and the p^{th} reward obstacle function $\Psi^{(p)}$ has the following form:

$$\Psi^{(p)}(\tau,S)=\begin{cases}\phi(T-\tau,S)+w^{\tau,(p-1)}(\delta,M,N) & \text{for } \tau\in[\delta,T]\\ \phi(T-\tau,S) & \text{for } \tau\in[0,\delta).\end{cases} \tag{15}$$

In (15), $w^{\tau,(p-1)}(\delta,M,N)$ denotes the value of the swing option with one fewer exercise right. In order to obtain the value of $w^{\tau,(p-1)}(\delta,M,N)$ for $\tau\in[\delta,T]$, when $p=1$ we note that

$$w^{\tau,(0)}(t,M,N)=0 \quad \text{for } (t,M,N)\in[0,\delta]\times\mathbb{R}_+^2,$$

whereas when $p>1$ we need to solve the following PDE problem:

$$\begin{aligned}\mathcal{L}[w^{\tau,(p-1)}]&=0 &&\text{in } (0,\delta)\times\mathbb{R}_+^2,\\ w^{\tau,(p-1)}(0,\cdot)&=u^{(p-1)}(\tau-\delta,\cdot) &&\text{in } \mathbb{R}_+^2,\end{aligned} \tag{16}$$

where $\mathcal{L}$ is given by (14).

Note that due to the constant refracting period, the reward function (15) can be equivalently written as

$$\Psi^{(p)}(\tau,S)=\begin{cases}\phi(T-\tau,S)+w^{\tau,(p-1)}(\delta,M,N) & \text{for } \tau\geq(p-1)\delta\\ \Psi^{(p-1)}(\tau,S) & \text{for } \tau<(p-1)\delta.\end{cases} \tag{17}$$

That is, in a period of length $(p-1)\delta$ we only can exercise $(p-1)$ rights due to the refracting period. That is why the value of the reward function with p exercise rights is equal to the value with $(p-1)$ rights at any time $\tau<(p-1)\delta$.

3 Numerical methods

As in the case without jumps, in order to obtain a numerical approximation of the value of a swing option with $p\in\mathbb{N}$ exercise rights, we need to solve a free boundary problem for each value of p. Additionally, for $p>1$, in order to obtain the value of the reward function, $\Psi^{(p)}(\tau,S)$, associated with each complementarity problem (13), the solution for certain times of an initial value problem is required. The difference is that the problems are linked to an integro-differential operator. For the numerical solution of the PIDEs (13) and (16), we propose a Crank-Nicolson characteristics time discretization scheme combined with a piecewise quadratic Lagrange finite element method and we treat explicitly in time the integral term, which is discretized with a suitable quadrature formula. For the additional inequality constraints associated with the complementarity problem (13), we propose a mixed formulation and an Augmented Lagrangian Active Set (ALAS) technique.

3.1 Localization procedure and formulation in a bounded domain

In a similar way to the case without spikes, first, we need to approximate the unbounded domain in which the PIDE is posed, by a bounded one and we impose boundary conditions on the boundaries where they are required. Moreover, we localize the domain of integration in the integral term that appears with the presence of jumps in the electricity price. Let us introduce the notation:

$$x_1 = M, \quad x_2 = N \quad \text{and} \quad \bar{x}_2 = \ln(x_2), \tag{18}$$

and let us consider both x_1^∞ and x_2^∞ to be large enough suitably chosen real numbers. Let

$$\Omega = (0, x_1^\infty) \times (0, x_2^\infty).$$

Then, let us denote the Lipschitz boundary by $\Gamma = \partial\Omega$ such that $\Gamma = \bigcup_{i=1}^{2}(\Gamma_i^- \cup \Gamma_i^+)$, where

$$\Gamma_i^- = \{(x_1, x_2) \in \Gamma \mid x_i = 0\}, \quad \Gamma_i^+ = \{(x_1, x_2) \in \Gamma \mid x_i = x_i^\infty\}, \quad i = 1, 2.$$

Following [28] which is based on the theory proposed by Fichera in [14] and as it is said in [6], we need to impose a boundary condition on Γ_1^+. On this boundary we impose the artificial boundary condition (ABC) derived in [6], taking into account the approach of [15]. After the previous change of spatial variables we write the equation (16) in divergence form in the bounded spatial domain $\Omega = (0, x_1^\infty) \times (0, x_2^\infty)$. Thus, the IBVP takes the following form:

Find $w^{\tau,(p-1)} : [0, \delta] \times \Omega \to \mathbb{R}$ such that

$$\frac{\partial w^{\tau,(p-1)}}{\partial t} + \vec{v} \cdot \nabla w^{\tau,(p-1)} - Div(A \nabla w^{\tau,(p-1)}) + l w^{\tau,(p-1)}$$
$$-\lambda \int_{z_{min}}^{z_{max}} \bar{w}^{\tau,(p-1)}(\tau, x_1, \bar{x}_2 + z)\nu(z)dz = \tilde{f} \qquad \text{in } (0, \delta) \times \Omega\,, \tag{19}$$
$$w^{\tau,(p-1)}(0, .) = u^{(p-1)}(\tau - \delta, .) \qquad \text{in } \Omega, \tag{20}$$
$$\frac{\partial w^{\tau,(p-1)}}{\partial t} + b\frac{\partial w^{\tau,(p-1)}}{\partial x_1} + c w^{\tau,(p-1)} = 0 \qquad \text{on } (0, \delta) \times \Gamma_1^+\,, \tag{21}$$

where $\bar{w}^{\tau,(p-1)}(\tau, x_1, \bar{x}_2 + z) = w^{\tau,(p-1)}(\tau, x_1, \exp(\bar{x}_2 + z))$, $b = \alpha\,(\ln(x_1^\infty) - \mu)\, x_1^\infty$ and $c = l$. Furthermore, for the complementarity problem associated with the swing option value, denoting by P the Lagrange multiplier, we can pose the mixed formulation:

Find $u^{(p)} : [0, T] \times \Omega \to \mathbb{R}$ such that

$$\frac{\partial u^{(p)}}{\partial \tau} + \vec{v} \cdot \nabla u^{(p)} - Div(A \nabla u^{(p)}) + l u^{(p)}$$
$$-\lambda \int_{z_{min}}^{z_{max}} \bar{u}^{(p)}(\tau, x_1, \bar{x}_2 + z)\nu(z)dz + P = \tilde{f} \qquad \text{in } (0, T) \times \Omega\,, \tag{22}$$

with the complementarity conditions

$$u^{(p)} \geq \Psi^{(p)}, \quad P \leq 0, \quad (u^{(p)} - \Psi^{(p)})P = 0 \quad \text{in } (0, T) \times \Omega \tag{23}$$

where $\bar{u}^{(p)}(\tau, x_1, \bar{x}_2 + z) = u^{(p)}(\tau, x_1, \exp(\bar{x}_2 + z))$ and we consider the initial and boundary conditions

$$u^{(p)}(0, .) = \Psi^{(p)}(0, .) \qquad \text{in } \Omega, \tag{24}$$
$$\frac{\partial u^{(p)}}{\partial \tau} + b\frac{\partial u^{(p)}}{\partial x_1} + c u^{(p)} = 0 \qquad \text{on } (0, T) \times \Gamma_1^+\,. \tag{25}$$

with $b = \alpha\,(\ln(x_1^\infty) - \mu)\,x_1^\infty$ and $c = l$. For both problems, the involved data is defined as follows:

$$\begin{aligned} A &= \begin{pmatrix} \frac{1}{2}\sigma^2 x_1^2 & 0 \\ 0 & 0 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} \tilde{g}(\tau, x_1) \\ \tilde{h}(x_2) \end{pmatrix}, \quad l = r, \quad \tilde{f} = 0, \\ \tilde{g}(\tau, x_1) &= \begin{cases} 0 & \text{if } x_1 = 0 \\ (\sigma^2 - \alpha\,(\mu(T-\tau) - \ln(x_1)))\,x_1 & \text{if } x_1 \neq 0, \end{cases} \\ \tilde{h}(x_2) &= \begin{cases} 0 & \text{if } x_2 = 0 \\ (\beta \ln(x_2) + \lambda\kappa) x_2 & \text{if } x_2 \neq 0. \end{cases} \end{aligned}$$

Remark 3.1 *Note that the differential part of the PIDE is defined in the domain* $[0, x_1^\infty] \times [0, x_2^\infty]$*, using the discrete grid* $0 = x_{2_0}, x_{2_1}, ..., x_{2_q} = x_2^\infty$*. Since* $\ln(x_{2_0}) = -\infty$*, we choose* $z_{min} = \ln(x_{2_1})$ *and* $z_{max} = \ln(x_{2_q})$ *as it is proposed in [13].*

3.2 Time discretization

The method of characteristics is based on a finite differences scheme for the discretization of the material derivative. The material derivative is given by:

$$\frac{DF}{D\tau} = \frac{\partial F}{\partial \tau} + \vec{v} \cdot \nabla F. \tag{26}$$

First, we define the characteristics curve through $\mathbf{x} = (x_1, x_2)$ at time $\bar{\tau}$, $X(\mathbf{x}, \bar{\tau}; s)$, which satisfies:

$$\frac{\partial}{\partial s} X(\mathbf{x}, \bar{\tau}; s) = \vec{v}(X(\mathbf{x}, \bar{\tau}; s)), \quad X(\mathbf{x}, \bar{\tau}; \bar{\tau}) = \mathbf{x}. \tag{27}$$

In order to discretize in time the material derivative in the complementarity problem (22), let us consider a number of time steps $\bar{N}$, the time step $\Delta\tau = T/\bar{N}$ and the time mesh points $\tau^n = n\Delta\tau$, $n = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots, \bar{N}$. In order to obtain the initial condition for solving the problem (19) the time discretization has to be chosen such that $\delta/\Delta\tau \in \mathbb{N}$. So, we should choose $\bar{N}$ as a multiple of T/δ. In the discretization of the material derivative in the initial value problem (19) we consider a number of time steps equal to $\delta/\Delta\tau$.

The material derivative approximation by the characteristics method for both problems is given by:

$$\frac{DF}{D\tau} = \frac{F^{n+1} - F^n \circ X^n}{\Delta\tau},$$

where $F = u^{(p)}, w^{\tau,(p-1)}$ and $X^n(\mathbf{x}) := X(\mathbf{x}, \tau^{n+1}; \tau^n)$. For the case of $f = 0$ (i.e., without seasonality effect), the components of $X^n(\mathbf{x})$ can be computed analytically:

$$\begin{aligned} X_1^n(\mathbf{x}) &= \begin{cases} x_1 & \text{if } x_1 = 0 \\ \exp\left[(\exp(-\alpha\Delta\tau)(\sigma^2 + \alpha\ln(x_1)) - \sigma^2)/\alpha\right] & \text{if } x_1 \neq 0, \end{cases} \\ X_2^n(\mathbf{x}) &= \begin{cases} x_2 & \text{if } x_2 = 0 \\ \exp[((\beta\ln(x_2) + \lambda\kappa)\exp(-\beta\Delta\tau) - \lambda\kappa)/\beta] & \text{if } x_2 \neq 0. \end{cases} \end{aligned}$$

However, for the general case where it is not possible to compute the characteristics curves analytically, some numerical ODE solvers can be used (see [2], for example).

Next, we consider a Crank-Nicolson method around $\big(X(\mathbf{x},\tau^{n+1};\tau),\tau\big)$ for $\tau=\tau^{n+\frac{1}{2}}$ for the discretization in time of the differential part of the operator and we treat explicitly the integral term by using the Adams-Bashforth (AB) scheme proposed in [31]. Note that for the first time step, the AB scheme is reduced to the explicit scheme proposed in [9]. So, the time discretized equation for $F=u^{(p)}, w^{\tau,(p-1)}$ and $P=0$ can be written as follows:

Find F^{n+1} such that:

$$\begin{aligned}
&\frac{F^{n+1}(\mathbf{x})-F^n(X^n(\mathbf{x}))}{\Delta\tau}-\frac{1}{2}Div(A\nabla F^{n+1})(\mathbf{x})-\frac{1}{2}Div(A\nabla F^{n})(X^n(\mathbf{x}))\\
&+\frac{1}{2}(l\,F^{n+1})(\mathbf{x})+\frac{1}{2}(l\,F^{n})(X^n(\mathbf{x}))-\lambda\int_{z_{min}}^{z_{max}}\frac{3\bar{F}^n(x_1,\bar{x}_2+z)-\bar{F}^{n-1}(x_1,\bar{x}_2+z)}{2}\nu(z)dz \quad = \quad 0.
\end{aligned} \tag{28}$$

where $\bar{F}^n(x_1,\bar{x}_2+z)=F^n(x_1,\exp\bar{x}_2+z)$ and $\bar{F}^{n-1}(x_1,\bar{x}_2+z)=F^{n-1}(x_1,\exp\bar{x}_2+z)$. Note that the integral term is evaluated at the previous time steps, so that the presence of a full matrix in the linear systems associated with the fully discretized problems is avoided.

Moreover, we also discretize the artificial boundary condition (25) on Γ_1^+ as follows:

$$\frac{F^{n+1}(\mathbf{x})-F^n(\hat{X}^n(\mathbf{x}))}{\Delta\tau}+\frac{1}{2}(c\,F^{n+1})(\mathbf{x})+\frac{1}{2}(c\,F^{n})(\hat{X}^n(\mathbf{x})) \quad = \quad 0 \tag{29}$$

where $\hat{X}^n(\mathbf{x})=(-b\Delta\tau+x_1,x_2)^T$ in the case of $f=0$.

Thus,

$$F^{n+1}(\mathbf{x})=\frac{1-c\Delta\tau/2}{1+c\Delta\tau/2}F^n(\hat{X}^n(\mathbf{x})) \quad on \quad \Gamma_1^+. \tag{30}$$

In order to obtain the variational formulation of the semi-discretized problem, we multiply (28) by a suitable test function, integrate in Ω, use the classical Green formula and the following one [27]:

$$\begin{aligned}
\int_\Omega Div(\mathbf{A}\nabla F^n)(X^n(\mathbf{x}))\psi(\mathbf{x})d\mathbf{x} \quad = \quad & \int_\Gamma(\nabla X^n)^{-T}(\mathbf{x})\mathbf{n}(x)\cdot(\mathbf{A}\nabla F^n)(X^n(\mathbf{x}))\psi(\mathbf{x})d\mathbf{x}\\
&-\int_\Omega(\nabla X^n)^{-1}(\mathbf{x})(\mathbf{A}\nabla F^n)(X^n(\mathbf{x}))\cdot\nabla\psi(\mathbf{x})d\mathbf{x}\\
&-\int_\Omega Div((\nabla X^n)^{-T}(\mathbf{x}))(\mathbf{A}\nabla F^n)(X^n(\mathbf{x}))\psi(\mathbf{x})d\mathbf{x}
\end{aligned} \tag{31}$$

Note that, when $f=0$, we have:

$$Div((\nabla X^n)^{-T}(\mathbf{x}))=\begin{pmatrix}\dfrac{1}{e_1}(\exp(\alpha\Delta\tau)-1)\\ \\ \dfrac{1}{e_2}(\exp(\beta\Delta\tau)-1)\end{pmatrix}, \tag{32}$$

where $e_1 = \exp\left[(\exp(-\alpha\Delta\tau)(\sigma^2 + \alpha\ln(x_1)) - \sigma^2)/\alpha\right]$ and $e_2 = \exp[((\beta\ln(x_2) + \lambda\kappa)\exp(-\beta\Delta\tau) - \lambda\kappa)/\beta]$. In the general case $Div((\nabla X^n)^{-T}(\mathbf{x}))$ needs to be approximated. After the previous steps, we can write a variational formulation for the time discretized problem as follows:

Find $F^{n+1} \in H^1(\Omega)$ such that, for all $\psi \in H^1(\Omega)$ such that $\psi = 0$ on Γ_1^+:

$$\int_\Omega F^{n+1}(\mathbf{x})\psi(\mathbf{x})d\mathbf{x} + \frac{\Delta\tau}{2}\int_\Omega (\mathbf{A}\nabla F^{n+1})(\mathbf{x})\nabla\psi(\mathbf{x})d\mathbf{x} + \frac{\Delta\tau}{2}\int_\Omega lF^{n+1}(\mathbf{x})\psi(\mathbf{x})d\mathbf{x}$$
$$= \int_\Omega F^n(X^n(\mathbf{x}))\psi(\mathbf{x})d\mathbf{x} - \frac{\Delta\tau}{2}\int_\Omega (\nabla X^n)^{-1}(\mathbf{x})(\mathbf{A}\nabla F^n)(X^n(\mathbf{x}))\nabla\psi(\mathbf{x})d\mathbf{x}$$
$$-\frac{\Delta\tau}{2}\int_\Omega lF^n(X^n(\mathbf{x}))\psi(\mathbf{x})d\mathbf{x} + \lambda\Delta\tau\int_\Omega\left[\int_{z_{min}}^{z_{max}}\frac{3\bar{F}^n(x_1,\bar{x}_2+z) - \bar{F}^{n-1}(x_1,\bar{x}_2+z)}{2}\nu(z)dz\right]\psi(\mathbf{x})d\mathbf{x}$$
$$-\frac{\Delta\tau}{2}\int_\Omega Div((\nabla X^n)^{-T}(\mathbf{x}))(\mathbf{A}\nabla F^n)(X^n(\mathbf{x}))\psi(\mathbf{x})d\mathbf{x}, \quad (33)$$

where ∇X^n can be computed analytically in some cases. Otherwise it needs to be numerically approximated (see [2], for example).

3.3 Finite elements discretization and nonlinear terms

For the spatial discretization we consider piecewise biquadratic Lagrange finite elements. For the numerical integration of the terms appearing in this finite elements discretization, we use a 9-points quadrature formula which implies a lumped mass matrix computation when dealing with this term.

In order to deal with the nonlinearities in the free boundary problem that governs the valuation of the swing options, we apply to the mixed formulation (22)-(23) the ALAS algorithm proposed in [19] and explained in detail for the case of swing options without spikes in the electricity price in [6]. This algorithm is applied to the fully discretized in time and space problem and computes sequences that converge to the swing option value, to the Lagrange multiplier associated with the inequality constraint and to the early exercise and non-early exercise regions, thus allowing also to identify an approximation to the optimal exercise boundary.

3.4 Approximation of the integral term

In order to approximate the integral term that appears in the PIDE due to the presence of jumps in the electricity price, we use a suitable numerical integration procedure. More precisely, we use the classical composite trapezoidal rule with $m+1$ points in the following way:

$$\int_{z_{min}}^{z_{max}} G(x_1,\bar{x}_2+z)\nu(z)dz \approx \frac{h}{2}\left[G(x_1,\bar{x}_2+z_{min})\nu(z_{min}) + G(x_1,\bar{x}_2+z_{max})\nu(z_{max}) + 2\sum_{j=1}^{m-1} G(x_1,\bar{x}_2+k_j)\nu(k_j)\right]$$

where $G(x_1, \bar{x}_2 + z) = \frac{3\bar{F}^n(x_1,\bar{x}_2+z)-\bar{F}^{n-1}(x_1,\bar{x}_2+z)}{2}$, $h = \frac{z_{max}-z_{min}}{m}$, $k_j = z_{min} + jh$ for $j = 1, ..., m-1$.

4 Numerical results

In this section we show some numerical results to illustrate the performance of the numerical methods, by comparing them with some examples in the literature with and without the presence of jumps in the electricity price. Note that this paper is the first one to consider the numerical solution of the PIDE associated with a two factor model for electricity prices. Thus, we mainly compare our results with the example in [16] considering two factors and a binomial method.

4.1 Example 1

In this example we consider as in [16] that the dynamics of the stochastic process N_t under a risk neutral probability measure is given in terms of the Poisson process, $\tilde{Z}$, instead of the compensated one, in the following way:

$$dN_t \quad = \quad -\beta \ln(N_t) N_t dt + (\exp(J_t) - 1) N_t d\tilde{Z}_t \,. \tag{34}$$

Moreover, we assume that $J \sim \exp(1/\mu_j)$ with density function

$$\nu(z) = \frac{1}{\mu_j} \exp\left(-\frac{1}{\mu_j} z\right) \mathbb{1}_{z \geq 0}.$$

Thus, the integro-differential operator in (14) is written as

$$\begin{aligned}\mathcal{L}[F] = -\frac{\partial F}{\partial \tau} + \frac{1}{2}\sigma^2 M^2 \frac{\partial^2 F}{\partial M^2} + \alpha\left(\mu(T-\tau) - \ln(M)\right) M \frac{\partial F}{\partial M} - \beta \ln(N) N \frac{\partial F}{\partial N} \\ -(r+\lambda)F + \lambda \int_0^\infty F(\tau, M, N\exp(z))\nu(z)dz.\end{aligned} \tag{35}$$

In the present example, we consider as in [16] the valuation of a swing option with up to $p = 20$ exercise rights where the rights correspond to the payoff of a call option. Moreover, we take into account the cases with spikes ($\lambda = 4$) or without spikes ($\lambda = 0$) in the electricity price. For this purpose, we need to specify a set of parameters, related to the market values of the data involved in the underlying factors, the initial conditions of the stochastic processes and the parameters of the payoff function. All of them are taken from [16] and are shown in Table 1. We have chosen these parameters in order to compare the results we obtain with the ones in [16] where an alternative binomial method has been used. Moreover, concerning the numerical methods we select the parameters collected in Table 2. Note that, as we consider $f = 0$, thus neglecting seasonality, b does not depend on time in this particular example.

In Figure 1 we show the value per exercise right with and without spikes of the swing option when the maturity of the contract is one year and a right can be exercised at most once per

Market parameters of the underlying factors	
Speed of mean reversion process M, α	7
Volatility, σ	1.4
Speed of mean reversion process N, β	200
Interest rate, r	0
Seasonality, f	0
Parameter of Poisson process, λ	0, 4
Parameter of jump size distribution, μ_j	0.4
Initial conditions	
Initial value of M, M_0	1
Initial value of N, N_0	1
Payoff function parameters	
Payoff, $\phi(T, S)$	$(S-K)_+$
Strike, K	1

Table 1: Fixed parameters of the model for Example 1, cf. [16].

Computational domain	
x_1^∞	4K
x_2^∞	4K
ABC	
Coefficient b	20.13
Finite elements mesh data	
Number of elements	576
Number of nodes	2401
ALAS algorithm	
Parameter γ	10000

Table 2: Parameters of the numerical methods in Example 1.

day (i.e. the refracting period δ is one day). Moreover, we consider that the time step $\Delta\tau$ is also one day.

Next, in Figure 2 we present the value per exercise right (with and without spikes) of the swing option when the maturity of the contract is two months, the refracting period δ is one day and the time step $\Delta\tau$ coincides with the refracting period. Finally, in Figure 3 we consider that the option has ten exercise opportunities per day (i.e. the refracting period is 0.1 days) and that the delivery period is six days. The time step $\Delta\tau$ is equal to the refracting period.

Figures 1, 2 and 3 are in full agreement with the analogous ones appearing in [16] and [21] obtained with binomial methods. More precisely, the results in Figure 1 agree with those in [16], Figure 10 (see also [21], Figure 4.8); the results in Figure 2 are the same as those in

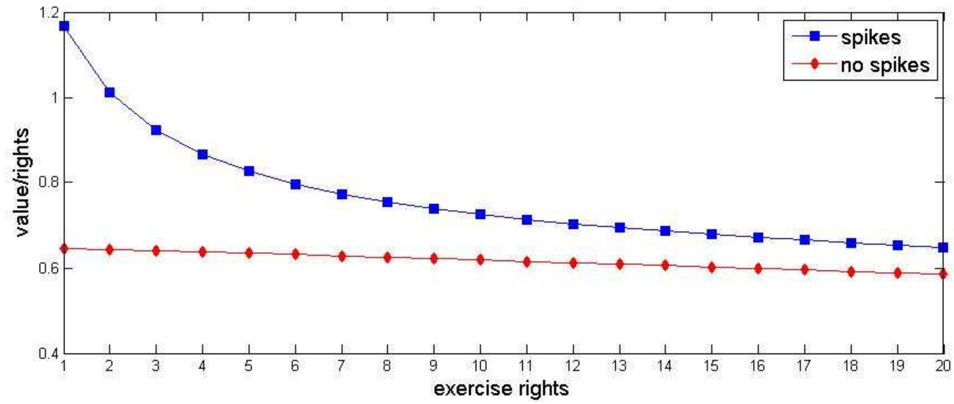

Figure 1: Value per right of a swing option with 1 year to delivery in Example 1.

the bottom-right graph of [16], Figure 11 (equivalently in [21], Figure 4.9); and our results in Figure 3 agree with those in [21], Figure 4.11. Furthermore, we can observe how the price per exercise right (with and without spikes) decreases with the number of exercise rights. It is what it is expected because p swing options with one exercise right (that would be equivalent to p American options) give more flexibility because you can exercise all the rights at once and consequently its price must be higher than the price of one swing option with p exercise rights. In Figure 3, the difference between two values per exercise right (with and without spikes) is smaller due to the value of the refracting period. As expected, when the value of the refracting period decreases the value of a swing option with p exercise rights tends to the value of p American options with 1 exercise right. In the presence of jumps the value of the swing option is higher than without spikes because the risk increases and the difference between both values is more significant for small numbers of exercise rights. According to [16], this is due to the option with more exercise rights will be used to protect from high prices due to the diffusive part and less due to the jump part.

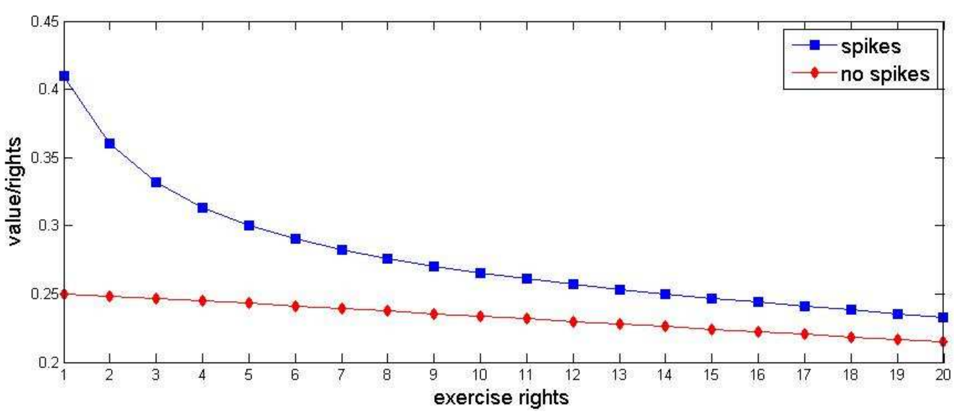

Figure 2: Value per right of a swing option with 60 days to delivery in Example 1.

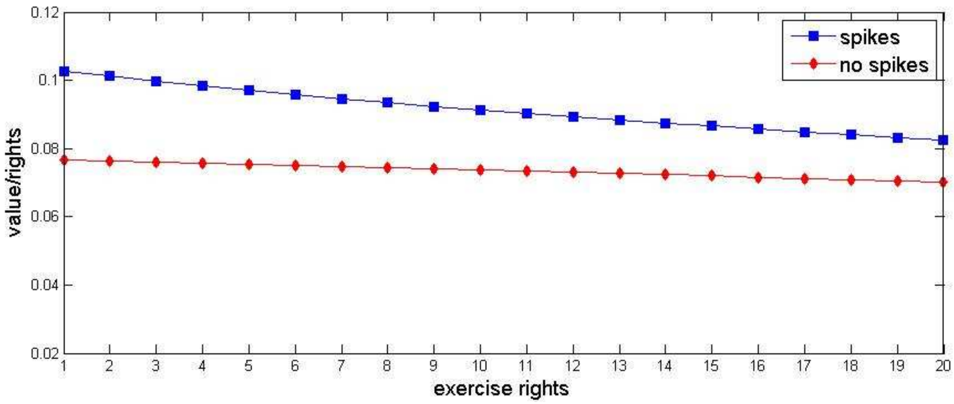

Figure 3: Value per right of a swing option with 6 days to delivery in Example 1.

4.2 Example 2

In this section, unlike Example 1, we show some cases in which the seasonality function and the interest rate are different from zero and we consider the possibility of jumps in the electricity price following Merton and Kou models. For this purpose, we consider a swing option with up to $p = 7$ rights, maturity 1 year and refracting period 0.1 years. Moreover, we consider the values for the parameters involved in the underlying factors which appear in Table 3. Most of them are taken from [34] for a one factor stochastic model for electricity prices, which in turn are taken from [23] and are obtained from experimental observations of daily electricity spot and future prices experimental observations. In order to pose a two factor model we consider different nonzero values for the parameter β. For the numerical solution we consider again the parameters in Table 2, except the coefficient b of the ABC, that in this case depends on time and it is always greater than zero. In this example, the time step is $\Delta\tau = 0.01$. Moreover, in order to compare the results obtained with Merton and Kou models we need a certain matching between the density functions of the normal distribution (Merton) and of the double-exponential distribution (Kou). For this purpose, we consider the parameters involved in the jump-diffusion models which are proposed in [13]. On one hand, in Figures 4 and 14 we show the value of this option per exercise right with and without spikes when $\beta = 0.2$ and $\beta = 2$, respectively, whereas on the other hand, in Figure 15 we represent its value for $\beta = 20$. Taking into account the three figures, we can observe that the value of the swing option, with and without spikes, decreases when we increase the value of the mean reversion parameter β. As it is indicated in [6] and according to [16], an increase in β implies a decrease in the asset value and, therefore, in the value of the call swing option. For the case of $\beta = 0.2$, in Figures 5 and 6 we represent the approximate location of the free boundary at origination (i.e. $t = 0$) when $p = 2$ and under Merton and Kou jump diffusion models, respectively. In addition, in Figure 7 we represent for the same case the free boundary with the possibility of spikes in the electricity price. For the three figures, in the white region it is optimal to exercise the option whereas the black region corresponds to the continuation region. Note that the presence of spikes does not affect two much the optimal exercise boundary. In Figures 8 and 9 we show the exercise value (or obstacle) at origination for the same swing contract and data set with spikes under following Merton or Kou models, respectively. Analogously, in Figure 10 we represent the exercise value at origination without spikes. In Figures 11, 12 and 13 the swing option value at origination is shown for the cases with spikes under Merton or Kou models and without spikes, respectively. Note that in the white region of Figures 5, 6 and 7, the value of the swing option in Figures 11, 12 and 13 coincides with the exercise value represented in Figures 8, 9 and 10, respectively.

Market parameters of the underlying factors	
Speed of mean reversion process M, α	0.016
Volatility, σ	0.086
Speed of mean reversion process N, β	0.2, 2, 20
Interest rate, r	0.05
Seasonality, f	$4.867 + 0.306\cos\left((t+0.836)\frac{2\pi}{365}\right)$
Parameter of Poisson process, λ	0.8
Mean of jump size (Merton), μ_j	-0.1
Standard deviation of jump size (Merton), γ_j	0.45
Probability of upward jump (Kou), p_k	0.3445
Parameter (Kou), α_1	3.0465
Parameter (Kou), α_2	3.0775
Initial conditions	
Initial value of M, M_0	1
Initial value of N, N_0	1.5
Payoff function parameters	
Payoff, $\phi(T,S)$	$(S-K)_+$
Strike, K	1

Table 3: Fixed parameters of the model with seasonality in Example 2.

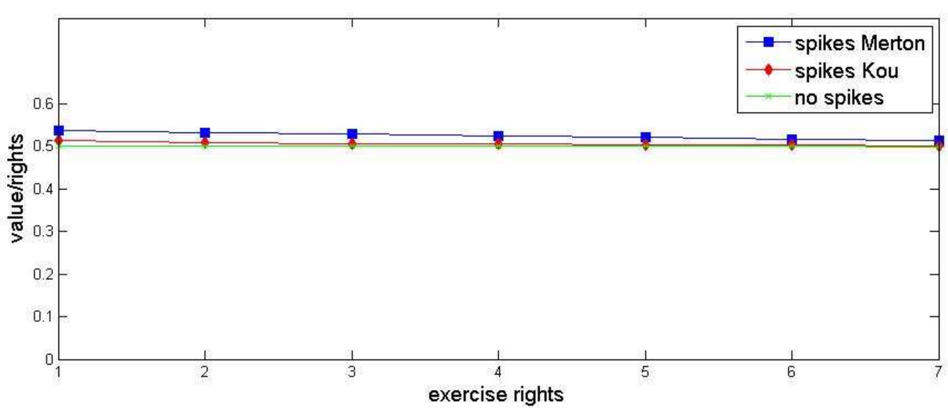

Figure 4: Value per right of a swing option with 1 year to delivery when $\beta = 0.2$ in Example 2.

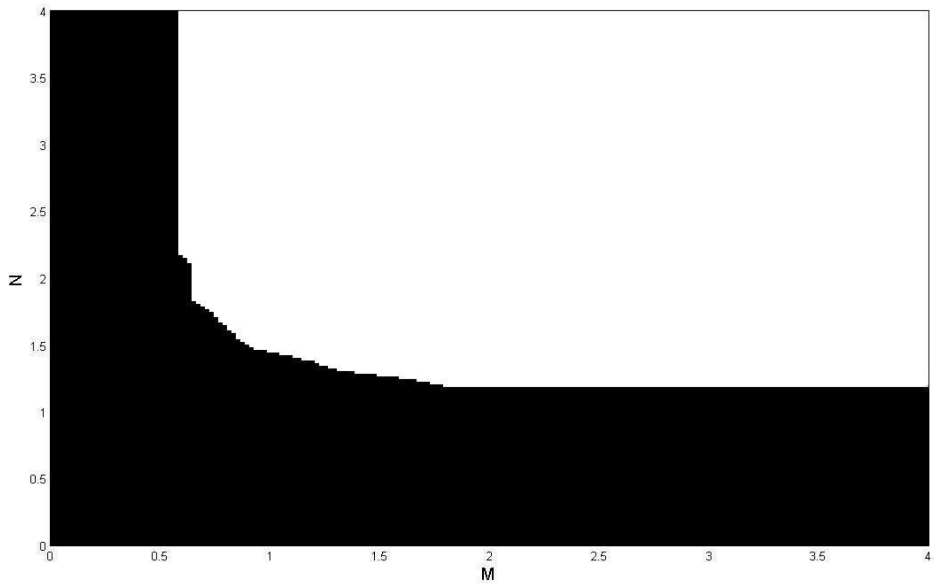

Figure 5: Approximated free boundary in the grid at origination of a swing option with p=2 rights in Example 2 when $\beta = 0.2$ and with spikes following Merton model. Exercise region in white and non exercise region in black.

5 Conclusions

In this paper we have considered the valuation of swing options in electricity markets where the electricity price dynamics is described by a jump-diffusion model. The assumption of jump-diffusion models seems reasonable to capture some features of the electricity prices observed in the markets. The consideration of spikes in the electricity prices leads to a PIDE problem, which has to be solved with specially designed numerical methods.

The swing option mainly consists of a path dependent option with multiple exercise rights. The right consists of receiving the payoff of a call option. The valuation problem has been posed as a sequence of free boundary problems associated with an integro-differential operator, one for each right. Additionally, an initial value problem linked to the same operator has to be solved due to the fact that the value of a swing option with one exercise less is involved in the definition of the obstacle function.

In order to obtain a numerical solution of the problem, we have proposed appropriate numerical methods based on Lagrange-Galerkin formulations combined with the ALAS algorithm to deal with the early exercise feature. Moreover, the integral term that arises due to the presence of jumps is explicitly treated. Finally, we show some numerical results in order to illustrate the behaviour of the proposed methods and the quantitative and qualitative properties of the solutions, as well as the difference in the swing value with and without spikes in the electricity prices. It is observed that the presence of spikes increases the value of the swing option although

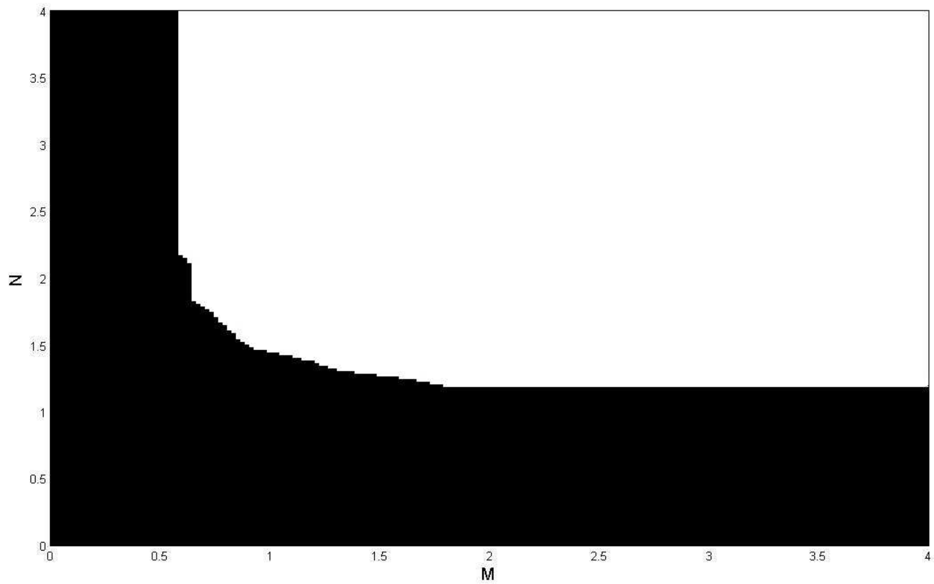

Figure 6: Approximated free boundary in the grid at origination of a swing option with p=2 rights in Example 2 when $\beta = 0.2$ and with spikes following Kou model. Exercise region in white and non exercise region in black.

it does not affect too much the optimal exercise boundary.

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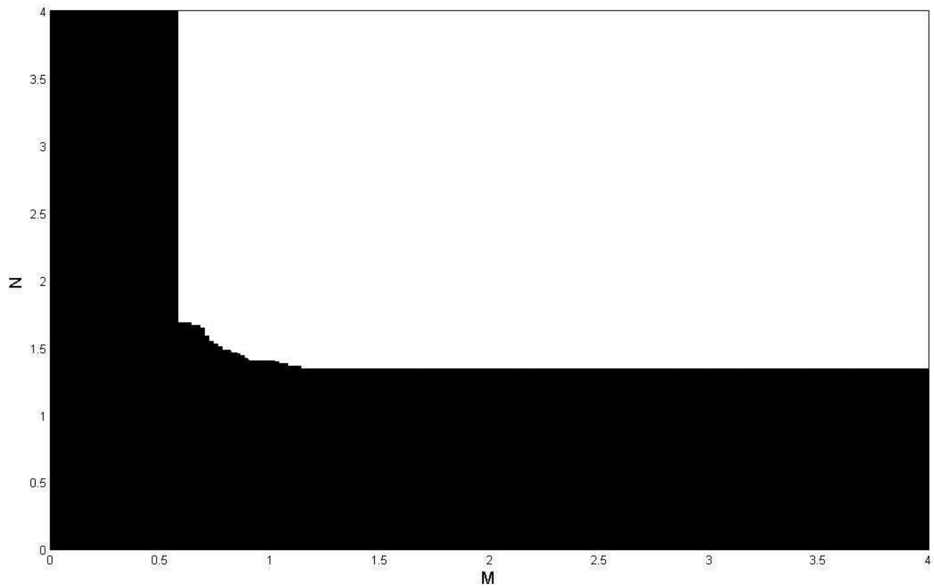

Figure 7: Approximated free boundary in the grid at origination of a swing option with p=2 rights in Example 2 when $\beta = 0.2$ and without spikes. Exercise region in white and non exercise region in black.

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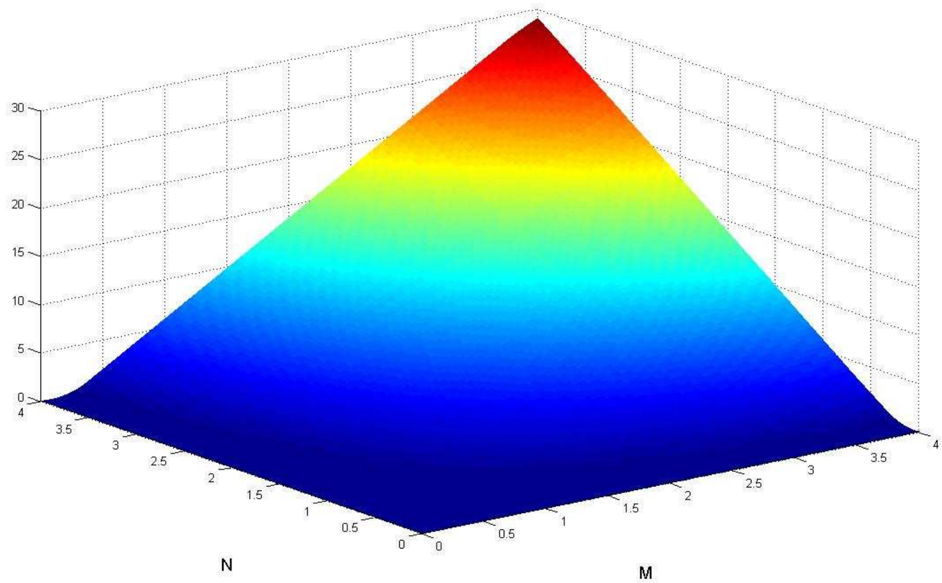

Figure 8: Obstacle at origination of a swing option with p=2 rights in Example 2 when $\beta = 0.2$ and with spikes following Merton model.

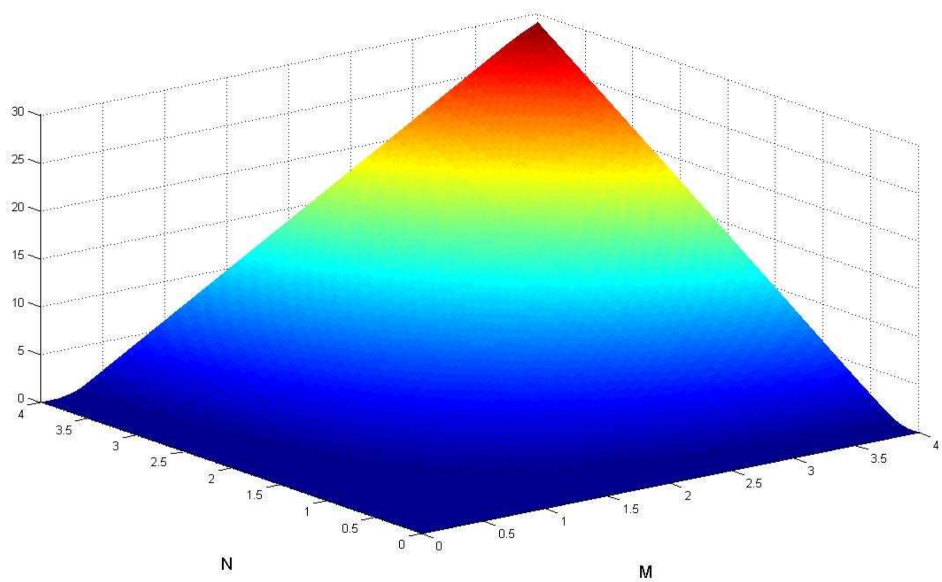

Figure 9: Obstacle at origination of a swing option with p=2 rights in Example 2 when $\beta = 0.2$ and with spikes following Kou model.

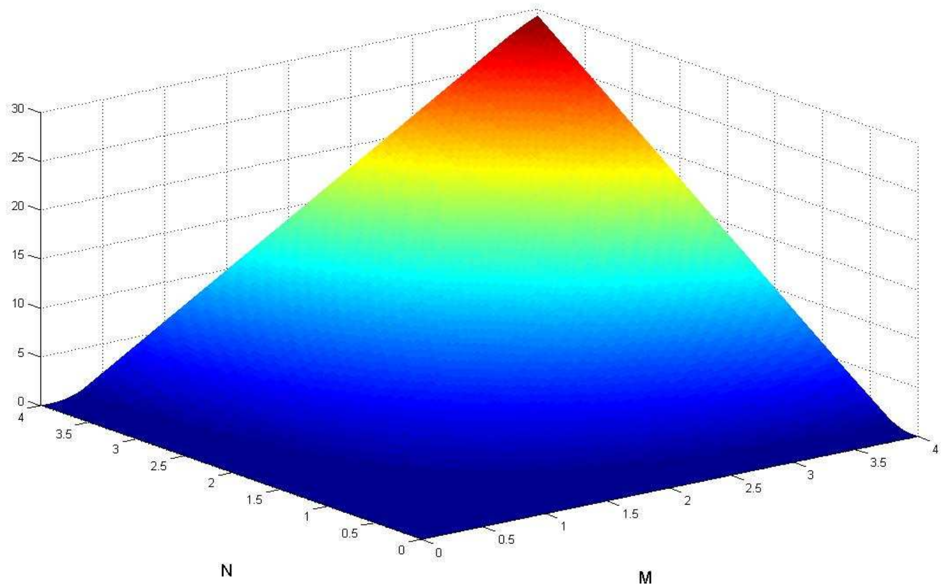

Figure 10: Obstacle at origination of a swing option with p=2 rights in Example 2 when $\beta = 0.2$ and without spikes.

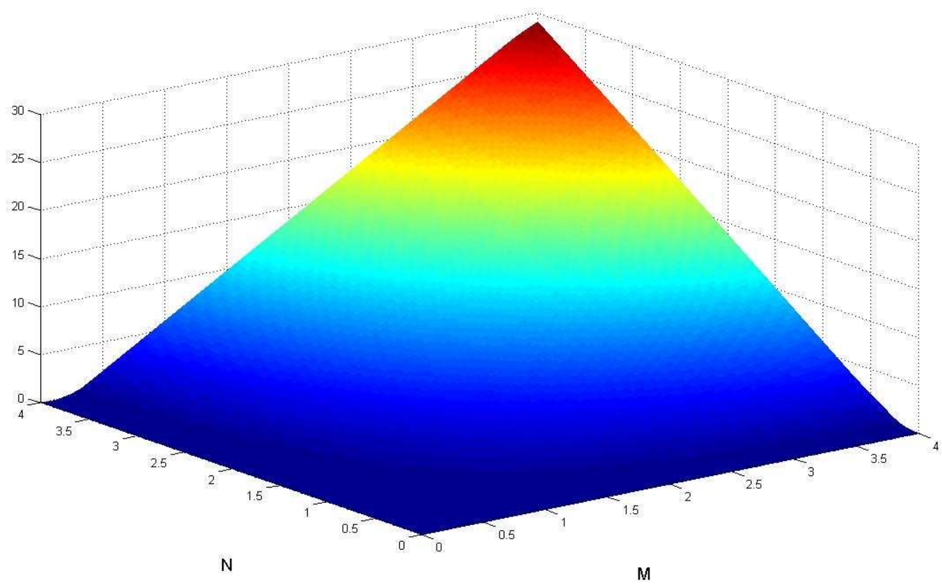

Figure 11: Swing option value at origination with p=2 rights in Example 2 when $\beta = 0.2$ and with spikes following Merton model

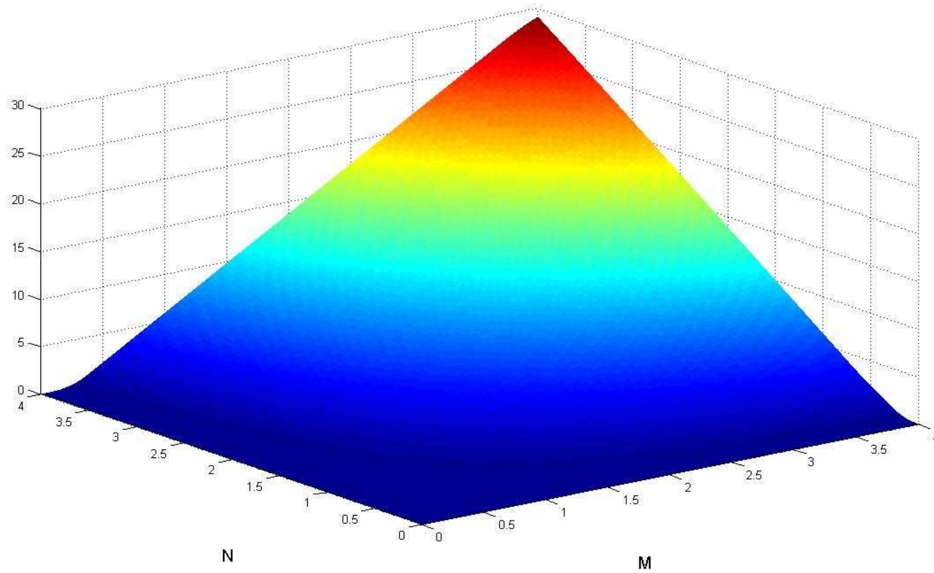

Figure 12: Swing option value at origination with p=2 rights in Example 2 when $\beta = 0.2$ and with spikes following Kou model

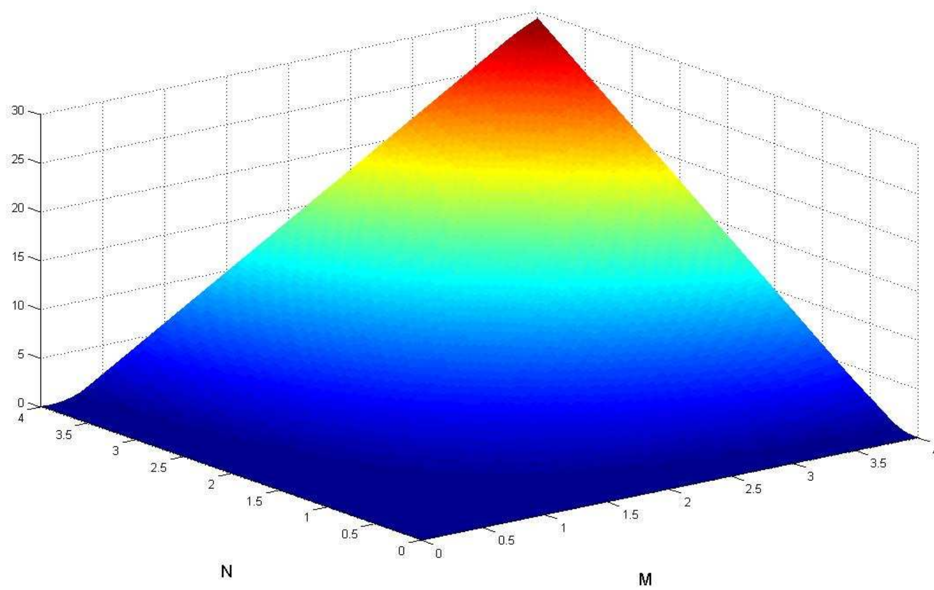

Figure 13: Swing option value at origination with p=2 rights in Example 2 when $\beta = 0.2$ and without spikes

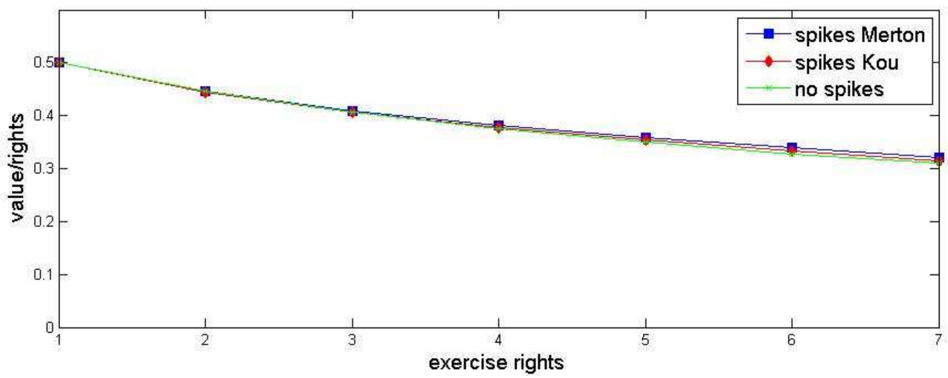

Figure 14: Value per right of a swing option with 1 year to delivery when $\beta = 2$ in Example 2.

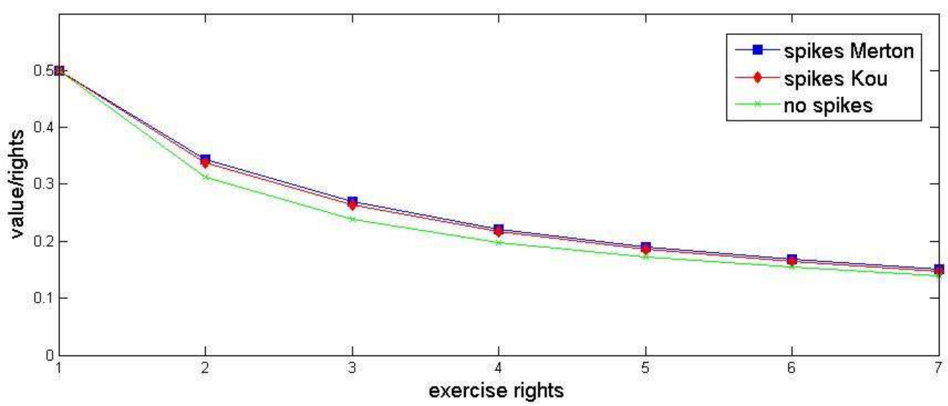

Figure 15: Value per right of a swing option with 1 year to delivery when $\beta = 20$ in Example 2.

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