Bergische Universität Wuppertal

Fachbereich Mathematik und Naturwissenschaften

Institute of Mathematical Modelling, Analysis and Computational Mathematics (IMACM)

Preprint BUW-IMACM 14/26

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August 27, 2014

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We present a probabilistic low cycle fatigue (LCF) Model for turbo charger compressor of a jet engine. This compressor is made of casted aluminum alloy (C355). This model has been implemented using the finite element method in R Language [4].

1 The Physical Model: The Linear Elasticity Equation

Let us consider a domain $\Omega \subset \mathbb{R}^3$ which represent a deformable medium like a turbo charger compressor. The function $f : \Omega \to \mathbb{R}^3$ represent the external load applied an Ω. Moreover let $u : \Omega \to \mathbb{R}^3$ the displacement field. The boundary of the compressor $\partial\Omega$ is partitioned into a clamped boundary $\partial\Omega_D$ and a Neumann boundary $\partial\Omega_N$. The normal load imposed on the surface boundary is described by the function $g : \partial\Omega \to \mathbb{R}^3$. According to [1], the following equation describes our linear isotropic elasticity model:

$$\begin{cases} \nabla.\sigma(u) + f = 0, & \text{in } \Omega \\ \sigma(u) = \lambda(\nabla.u)\mathcal{I} + \mu(\nabla u + \nabla u^T), & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega_D, \\ \sigma(u).n = g, & \text{on } \partial\Omega_N. \end{cases}$$

Here, λ and μ are the Lamé coefficients. The linearized strain rate tensor $\varepsilon(u) : \Omega \to \mathbb{R}^{3,3}$ is defined as: $\varepsilon(u) = \frac{1}{2}(\nabla u + \nabla u^T)$. Numerical solutions of this equation can be computed by a finite element approximation, confer [7].

2 The local and probabilistic model for LCF

We refer in this section to the LCF-model described in the work [2]. The fatigue process is modelled by a continuous time process, where the time is a random variable N describing the number of cycles to failure. The local and probabilistic model for LCF ist given by the cumulative distribution function:

$$F_N(n) = 1 - \exp\left(- \int_0^n \int_{\partial\Omega} \frac{m}{N_{det}} \left(\frac{s}{N_{det}} \right)^{m-1} dAds \right) = 1 - \exp\left(-J_{sur}(\Omega, u).n^m \right)$$

where m is the Weibull shape and $J_{sur}(\Omega, u)^{1/m}$ is the Weibull scale parameter which describes the number N of cycles to crack initiation.

3 Finite Element Approximation

In order to compute the probability of failure we need to apply the finite element method to evaluate the following surface integral:

$$J_{sur}(\Omega, u) = \int\limits_{\partial\Omega} \left(\frac{1}{N_{det}(\varepsilon(\nabla u(x)))} \right)^m dA.$$

For more details concerning the computation of N_{det}, we refer to [2].
Denote by $\mathcal{N}_h$ a collection of the boundary faces. The computation of surface integral $J_{sur}(\Omega, u)$ reduces to evaluating integrals over each element in the collection $\mathcal{N}_h$:

$$\int\limits_{\partial\Omega} \left(\frac{1}{N_{det}(\varepsilon^e(\nabla u(x)))} \right)^m dA \approx \sum_{F \in \mathcal{T}_h} \int\limits_F \left(\frac{1}{N_{det}(\varepsilon^e(\nabla u(x)))} \right)^m dA,$$

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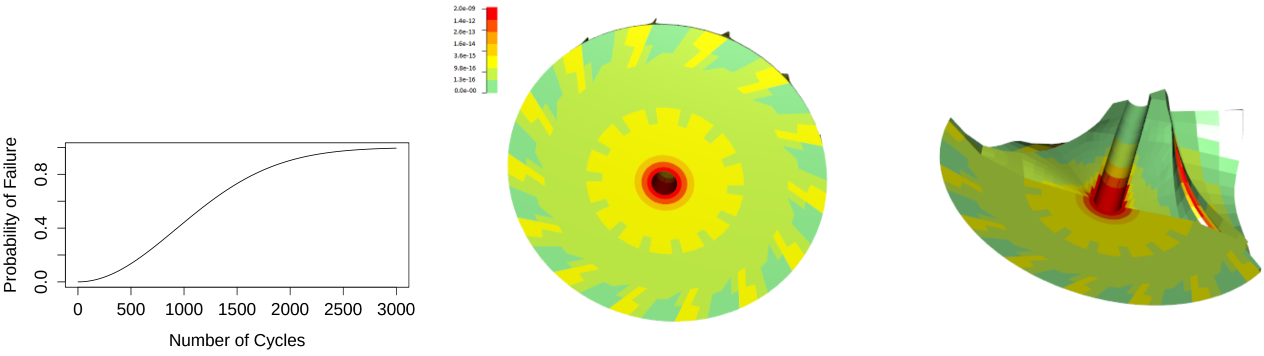

Fig. 1: The probability of failure and lifetime prediction for turbo charger compressor from different views: Red areas show critical regions

where ε^e is the elastic strain tensor.
According to [1] we denote by $T_F : \widehat{F} \to F$ a $\mathcal{C}^1$-diffeomorphism mapping the geometric reference face $\widehat{F} \subset \mathbb{R}^2$ to any face F in $\mathcal{N}_h$. Let $g_F(\widehat{x}) = (J_F(\widehat{x}))^T J_F(\widehat{x})$ be the metric tensor where $J_F(\widehat{x}) = \frac{\partial T_F(\widehat{x})}{\partial \widehat{x}}$ is the Jacobian matrix of the mapping T_F at $\widehat{x}$. The change of variables $x = T_K(\widehat{x})$ yields :

$$\int_F \left(\frac{1}{N_{det}(\varepsilon^e(\nabla u(x)))} \right)^m dA = \int_{\widehat{F}} \left(\frac{1}{N_{det}(\varepsilon^e(\nabla u(T_F(\widehat{x}))))} \right)^m \sqrt{det(g_F(\widehat{x}))} d\widehat{A}$$

By considering a quadrature on $\widehat{F}$ defined by l_q^∂ Gauss points $\{\widehat{\xi_1^\partial}, ..., \widehat{\xi_{l_q^\partial}^\partial}\}$ and l_q^∂ weights $\{\widehat{\omega_1^\partial}, ..., \widehat{\omega_{l_q^\partial}^\partial}\}$ and by setting $\omega_{lF} = \widehat{\omega}_l \sqrt{det(g_F(\widehat{\xi_l}))}$ and $\xi_{lF} = T_F(\widehat{\xi_l})$ we will have:

$$J_{sur}(\Omega, u) \approx \sum_{F \in \mathcal{T}_h} \sum_{l=1}^{l_q} \omega_{lF} [N_{det}(\varepsilon_\nu(\nabla u(\xi_{lF}))))]^{-m}.$$

4 Computations and Results

The discretized cost functional J_{sur} in the previous section is computed for a model jet engine radial turbo compressor obtained from [3]. The preprocessing data (coordinates, elements connectivity and displacements) are handled using the free FEA-Software CalculiX. The compressor is discretized using a 20 node hexahedral element with reduced quadratures (he20r) into 9464 elements. J_{sur} is calculated over all 6356 faces (rectangles) on the compressor boundary. We used for this computation a self implemented FEM-tool in R language. The material parameters have been taken from [5]. The Weibull shape parameter is set $m = 2$, which is a usual value for polycristalline metal. We followed the approach described in [6] to adapt the CMB-parameters to our compressor model because the size of specimens have a non-negligible influence on crack-initiation life.
Figure 1 shows the probability of failure of turbo charger compressor and the computed local crack formation intensity $(\frac{1}{N_{det}})^m$ over all faces on the boundary of the compressor.

Acknowledgements This work has been supported by the German federal ministry of economic affairs BMWi and the gasturbine technology department Siemens Energy via an AG Turbo grant.

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