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**Popular Support to Terrorist Organizations:
A Short-Term Prediction based on a
Dynamic Model applied to a Real Case**

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Chapter 1

POPULAR SUPPORT TO TERRORIST ORGANIZATIONS: A SHORT-TERM PREDICTION BASED ON A DYNAMIC MODEL APPLIED TO A REAL CASE

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1. Introduction

Popular support is an important enabler for radical violent organizations and it may be crucial for their survival. At the same time, extremist groups have also an impact in the societies where they are inserted, especially if those groups are engaged in violent activities. Social and behavioral scientists try to find clues about how that interaction may affect those people, either at the group or at the individual level, in order to foresee subsequent dynamics [1, 2, 3, 4].

Out of the social and behavioral fields, the model presented by Castillo-Chavez and Song [5] deals with similar processes from a mathematical modeling perspective. In that paper, the authors divide the total population into what they call the core, i.e., people belonging or supporting an extremist organization, objective or idea, and the non-core, usually larger than the previous one. At the same time, the core is divided into the people who are not fully committed yet, what they call the semi-fanatic population, and the fully fanatic people. They also assume that an individual may become more fanatic by contacting with people more fanatic than him/her and, at the same time, that individuals in the core may leave the group at a certain rate. With these assumptions in mind, they present a continuous model and its long term analysis. Other authors [6, 7, 8] consider a network version of the Castillo-Chavez and Song's model based upon a system of ordinary differential equations and also study its long-term dynamics.

In this chapter we apply the Castillo-Chavez and Song's model to the Basque Country citizens' attitude towards the terrorist organization ETA (Basque Fatherland and Liberty) after that organization declared the cease of its violent activity, in 2011 [9]. Of course, what we apply here is the version of the model related to people supporting the organization and not to the version related to people belonging to it. Our objective is to analyse any short term dynamics appearing after that event. To do so, we take data from the Euskobarometro survey [10, Table 20], one of the best-known independent opinion polls in the region, as well as demographic index. Then, according to those data, we divide the population into the sub populations appearing in the Castillo-Chavez and Song's model, and we fit the model parameters by least square techniques. After that, we are able to predict in the short term the quantitative evolution of the fanatic population, which in turn constitutes an estimation of the bulk of people able to become new ETA members in upcoming years.

This chapter is organised as follows. In Section 2, we retrieve and prepare the necessary data from Euskobarometro. In Section 3, we recall the Castillo-Chavez and Song's model, scale it in order to adapt to the data magnitudes and assign values to the demographic parameters. In Section 4 we fit the model with the data and predict the evolution of the sub populations over the next few years. Finally, Section 5 is devoted to conclusions.

2. The Data for the Model

The Euskobarometro [10] ("Basque-barometer") is a sociological statistical survey in the Basque Country. It is conducted by the Department of Political Science of the University of

the Basque Country and it is based on personal interviews at home, asking questions about the sociological current issues, including ETA.

The question #20 of the Euskobarometro asks about the attitude of the Basque population towards the ETA and divide the population depending on their answer into eight sub populations: Total support; Justification with criticism; Goals yes / Means no; Before yes / Not now; Indifferent; ETA scares; Total rejection; No answer. In order to fit these eight sub populations with the four Castillo-Chavez and Song's ones, we group them into the following ones:

- Total support towards ETA.
- Attitude of justification with criticism.
- Remote justification attitude.
- Remaining attitudes (indifference, rejection, etc.).

In Table 1, we show the percentages for every sub population since January 2011, when ETA declared the cease of violent activities [9]. Note that the first Euskobarometro after January 2011 was issued in May 2011.

Date	Total support	Justification with criticism	Remote justification	Remaining attitudes
May 2011	1	3	21	75
Dec 2011	1	2	25	72
May 2012	1	4	29	66
Dec 2012	1	2	25	72
May 2013	1	2	28	69

Table 1. Percentage of Basque people in each sub population, classified depending on their attitude towards ETA.

3. The Model

First, we recall the Castillo-Chavez and Song's model [5]. This model is given by the following nonlinear system of ordinary differential equations

$$\begin{aligned}
G'(t) &= \Lambda T(t) - \beta_1 G(t) \frac{C(t)}{T(t)} + \gamma_1 S(t) + \gamma_2 E(t) + \gamma_3 F(t) - \mu G(t), \\
S'(t) &= \beta_1 G(t) \frac{C(t)}{T(t)} - \beta_2 S(t) \frac{E(t)+F(t)}{C(t)} - \gamma_1 S(t) - \mu S(t), \\
E'(t) &= \beta_2 S(t) \frac{E(t)+F(t)}{C(t)} - \beta_3 E(t) \frac{F(t)}{C(t)} - \gamma_2 E(t) - \mu E(t), \\
F'(t) &= \beta_3 E(t) \frac{F(t)}{C(t)} - \gamma_3 F(t) - \mu F(t), \\
T(t) &= G(t) + C(t), \\
C(t) &= S(t) + E(t) + F(t).
\end{aligned} \tag{1}$$

In (1), $G(t)$ is the non-core population, $C(t)$, in turn, is the core population, which includes $S(t)$, $E(t)$ and $F(t)$,

- $S(t) + E(t)$ is the semi-fanatic sub population,
- $F(t)$ is the fanatic sub population, which includes individuals who are completely committed.

$T(t)$ encompasses the total population. Finally, Λ is the constant birth rate, μ is the constant death rate, β_i , $i = 1, 2, 3$ are the transmission rates and γ_i , $i = 1, 2, 3$ are the transition backward rates.

In Figure 1, we can see a flow diagram of the model.

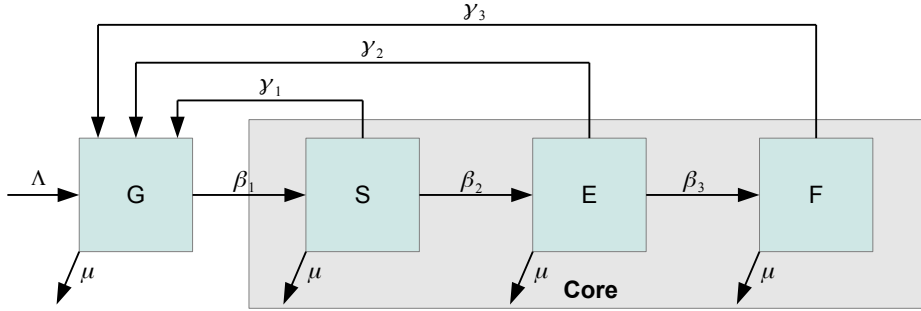


Figure 1. Model flow diagram. The arrows indicate the flow labelled by the corresponding parameters. It is an adaptation of Castillo-Chavez & Song model [5] for our purposes.

As said before, we identify Euskobarometro populations (see Table 1) with the model populations. Then,

- $F(t)$ will be those who have a total support attitude towards ETA.
- $E(t)$ will be the ones with an attitude of justification with criticism.
- $S(t)$ are those with an attitude of remote justification.

- $G(t)$ will be the remaining population.

Taking into account that the data in Table 1 is related to percentages meanwhile the model (1) is referred to number of individuals, we transform (scaling) the model into the same units as the data, because one of our objectives is to fit the data with the model in the next section.

Hence, following the ideas developed in the papers [11, 12] about how to scale models where the population is varying in size, we use the code described in [13] to scale the model. This process is very technical and does not provide relevant information, therefore, we are not going to describe here in detail and refer the interested reader to the references [11, 12, 13]. Furthermore, in the following, we are going to consider the populations $F(t)$, $E(t)$, $S(t)$ and $G(t)$ as the scaled ones.

In [14] we can find that the birth rate in the Basque Country in 2011 is $\Lambda = 0.00969$ and the mortality rate in the same year is $\mu = 0.00908$. We also consider the birth and death rates over the next years the same as the ones in 2011. The remaining model parameters $\beta_i, \gamma_i, i = 1, 2, 3$ are fitted with the data in Table 1.

4. Model fitting and prediction over the next few years

In order to compute the best fitting, we carried out computations with *Mathematica* [15] and we implemented the function

$$\begin{aligned} \mathbb{F} : \quad \mathbb{R}^6 & \rightarrow \mathbb{R} \\ (\beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3) & \mapsto \mathbb{F}(\beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3) \end{aligned}$$

such that:

1. Solve numerically (using *Mathematica* command `NDSolve[]`) the system of differential equations (1) with initial values given by the first row of Table 1,
2. For $t = \text{May 2011, Nov 2011, May 2012, Nov 2012, May 2013}$, evaluate the computed numerical solution for each sub population $F(t)$, $E(t)$, $S(t)$ and $G(t)$.
3. Compute the mean square error between the values obtained in Step 2 and the data in Table 1.

The function \mathbb{F} takes values in \mathbb{R}^6 and returns a positive real number. Hence, we minimize this function using the Nelder-Mead algorithm [16, 17], that does not need the computation of any derivative or gradient, which is impossible to know in this case. Thus, the values of $\beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3$ (all of them positive) that minimize the objective function \mathbb{F} are

$$\begin{aligned} \beta_1 &= 6.39902, & \beta_2 &= 1.03593 \times 10^{-9}, & \beta_3 &= 0.36436, \\ \gamma_1 &= 4.98113, & \gamma_2 &= 0.13285, & \gamma_3 &= 6.30120 \times 10^{-7}. \end{aligned} \quad (2)$$

The obtained model parameters (2) indicates that there is a large flow, entering and exiting, between populations G (Remaining attitudes) and S (Remote justification). Furthermore, the transition from S (Remote justification) to E (Justification with criticism) is

very difficult. Also, it is very difficult for the strongest supporters (F , the more "fanatics") to reconsider their position.

We substitute the fitted model parameters (2) into the scaled version of the model (1) and we calculate the output until May 2017. In Figure 2 we can see the prediction for the evolution of all the populations over the next few years. Numerical values in the dates of the coming eight Euskobarometro surveys are shown in Table 2.

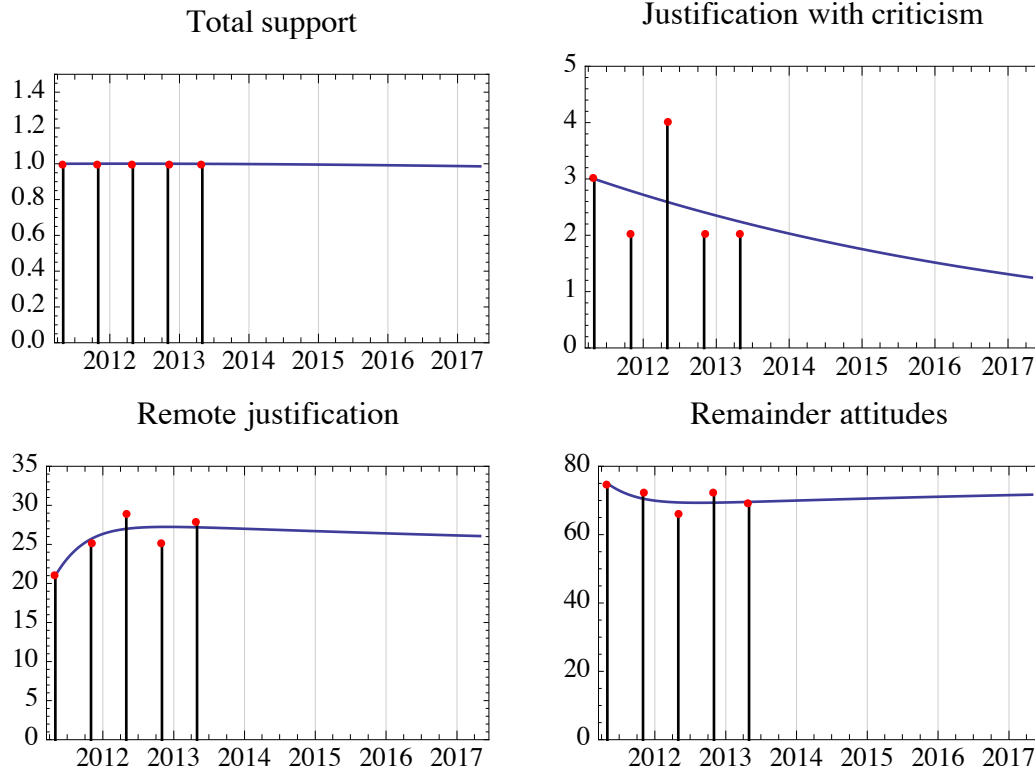


Figure 2. Model fitting (from May 2011 to May 2013) and prediction (from November 2013 to May 2017). Points are data in Table 1. The continuous line is the model output. Units are in percentages. Note that the scales for every graph are different. The decreasing in the population "Justification with criticism" is less than 2% from May 2011 to May 2017. The prediction is very stable over the next four years.

The prediction figures indicate a stabilization in the evolution of the attitudes towards ETA over the next few years, and therefore a stabilization in a hypothetical pool of candidates willing to join the organization in upcoming years.

5. Conclusions

In this chapter we applied the Castillo-Chavez and Song's model to a real situation where there is a significant impact of violent activities into the public opinion and vice-versa.

Date	Total support	Justification with criticism	Remote justification	Remaining attitudes
Nov 2013	0.999	2.080	27.050	69.870
May 2014	0.997	1.935	26.900	70.170
Nov 2014	0.996	1.797	26.750	70.460
May 2015	0.994	1.672	26.600	70.730
Nov 2015	0.992	1.553	26.460	71.000
May 2016	0.990	1.444	26.320	71.240
Nov 2016	0.988	1.342	26.190	71.480
May 2017	0.986	1.248	26.060	71.700

Table 2. Predicted percentage of Basque people in each sub population for the next eight Euskobarometro surveys, from May 2014 until May 2017. The predictions show a stable situation. The predicted variations over the next four years in each population are less than 1%.

To do so, we have divided the Basque population depending on their support attitude towards ETA, by using data series of the Euskobarometro, since January 2011, when ETA declared the cease of its violent activity. By using these data, we have developed an algorithm to find the model parameters that best fit the model with the data. Once the model has been calibrated, we use the obtained model parameters to predict the evolution of the different populations in the Basque Country over the next four years. As a result, the presented prediction states that the popular support to the ETA will remain stable, if and when the current scenario does not change.

However, as an epilogue, this might not be the case. In fact, the Spanish Ministry of Internal Affairs announced recently (Oct 27th, 2013) [18] that the Application no. 42750/09 of the European Court of Human Rights (Oct 21st, 2013) [19] will allow to release 50 members of the ETA from prison in two or three months. This notice constitutes an undoubted change in the present scenario, and therefore may provoke an impact in the above conclusions.

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