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Probabilistic Analysis of LCF Crack Initiation Life of a Turbine Blade under Thermomechanical Loading

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Abstract

An accurate assessment for fatigue damage as a function of activation and deactivation cycles is vital for the design of many engineering parts. In this paper we extend the probabilistic and local approach to this problem proposed in [1], [2] and [3] to the case of non-constant temperature fields and thermomechanical loading. The method has been implemented as a finite element postprocessor and applied to an example case of a gas-turbine blade which is made of a conventionally cast nickel base superalloy.

Keywords: Fatigue Crack Initiation; Probabilistic Fatigue; FE Analysis; Hazard Function;

1 Introduction

The necessity for a flexible service of a lot of engineering parts such as gas turbines leads to the importance of fatigue analysis, where probabilistic models can be very valuable. In this work, we present a probabilistic model for low-cycle fatigue (LCF) which can be derived from the Poisson point process or from a spatial hazard approach, confer [1] and [2], respectively. The model can be applied to polycrystalline metal which is sufficiently fine-grained so that isotropic material behavior can be assumed and continuum mechanics can be employed. Here, failure of a component is defined to be

Here, ε_a is an equivalent strain amplitude which can be derived from thermoelastic finite element analysis (FEA) with subsequent application of stress-strain relationships. For more details confer [3], [4] and [7].

Taking $F_N(n) = 1 - \exp\left(-\int_0^n h(s) ds\right)$ into account the probability of LCF crack initiation on the surface $\partial\Omega$ until cycle n is given by

$$F_N(n) = 1 - \exp\left(-\int_0^n \int_{\partial\Omega} \rho(s; \varepsilon_a, T) dA ds\right). \quad (3)$$

In Section 3 the hazard density function ρ will be employed to identify the critical and possibly overengineered regions of the component. Considering the statistical evaluations in [1] and following [2] and [8] we assume that the number N of cycles to crack initiation are Weibull distributed. Therefore, we choose the Weibull hazard ansatz

$$\rho(n; \varepsilon_a, T) = \frac{m}{N_{\det}(\varepsilon_a, T)} \left(\frac{n}{N_{\det}(\varepsilon_a, T)}\right)^{m-1}, \quad (4)$$

where m is the Weibull shape and $N_{\det}(\varepsilon_a, T)$ determines the corresponding Weibull scale parameter. If the Coffin-Manson-Basquin (CMB) equation – confer [4] and [6] – is taken for the strain-life relationship the scale field $N_{\det}(\mathbf{x}) = N_{\det}(\varepsilon_a(\mathbf{x}), T(\mathbf{x}))$ is given by the solution of

$$\varepsilon_a(\mathbf{x}) = \frac{\sigma'_f(T(\mathbf{x}))}{E(T(\mathbf{x}))} (2N_{\det}(\mathbf{x}))^{b(T(\mathbf{x}))} + \varepsilon'_f(T(\mathbf{x})) (2N_{\det}(\mathbf{x}))^{c(T(\mathbf{x}))} \quad (5)$$

on every point $\mathbf{x} \in \partial\Omega$, where the surface $\partial\Omega$ is subjected to an equivalent strain field $\varepsilon_a(\mathbf{x})$ and a temperature field $T(\mathbf{x})$. Having chosen an appropriate temperature model¹ for the CMB parameters $\sigma'_f, b, \varepsilon'_f, c$ and for Young's modulus E , the probabilistic model for LCF is given by the Weibull distribution

$$F_N(n) = 1 - \exp\left[-\left(\frac{n}{\eta}\right)^m\right] \quad \text{for scale} \quad \eta = \left(\int_{\partial\Omega} \frac{1}{N_{\det}^m} dA\right)^{-1/m} \quad (6)$$

and for some shape parameter $m \geq 1$, which yields the probability for LCF crack initiation to cycle n .

The model can be calibrated by means of usual maximum likelihood methods, confer [5] and [9], for example. The parameter m determines the scatter of the distribution where small values for $m \geq 1$ correspond to a large scatter.

The CMB parameters of the model are not the same as obtained from fitting standard specimen data. Due to the size effect, see [6], the original CMB approach leads to different values of the parameters for specimens under the same temperature and strain conditions but with different gauge

¹In this work we use a proprietary temperature model by Siemens AG.

areas. The new and more physical interpretation of these parameters – in context of the probabilistic model according to [1] and [2] – takes the size effect and inhomogeneous strain and temperature fields into account, so that the CMB parameters can be calibrated with LCF-test results of specimens with arbitrary geometry² and under arbitrary strain and temperature fields. Thus, these newly interpreted parameters can be assigned to every such geometry.

3 LCF Crack Initiation Life of a Turbine Blade

In this section we consider a turbine blade which is made of a polycrystalline cast nickel-based superalloy such as RENE 80. The following probabilistic analysis of its LCF crack initiation life is based on an FEA model for the operating state and on the Weibull distribution (6).

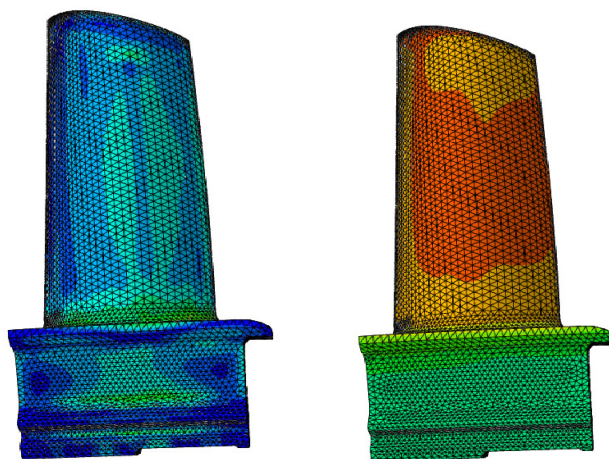


Figure 1: FEA results of Abaqus 6.9-2 for the von Mises stress field (left) and temperature field (right) of the turbine blade.

The temperature and strain field of the turbine blade in the operating state is computed by means of a thermoelastic FEA-model within Abaqus 6.9-2 and of the Neuber shakedown method³ which considers plasticity. The model includes approximately 190,000 tetrahedral, affine Lagrange elements of Serendipity class with 10 nodes. Figure 1 shows the von Mises stress and temperature field in the operating state. In the shutdown state the von Mises stress is everywhere zero and the temperature field is set equal to that one of the operating state which is an conservative approximation and avoids

²The specimen must consist of sufficiently many grains so that continuum mechanics can be applied. Moreover, information on when the first LCF crack initiation occurred has to be provided which can be practically difficult, however.

³Confer [4] and [7].

the treatment of thermo-mechanical fatigue (TMF). The transition from the shutdown state to the operating state and then back to the shutdown state is considered as one load cycle. It is further assumed that the shutdown and operating state stay the same during the cycles.

The probabilistic LCF model has been calibrated with strain controlled LCF test results⁴ for standard specimens which were subjected to different temperatures and strain amplitudes. The maximum likelihood method has been used for the calibration which constitutes a statistical estimation of the model parameters, confer [5]. This estimation in conjunction with computing the surface integral (6) results in a value for the Weibull scale parameter η which we call the maximum likelihood value for η .

As the estimation depends on the LCF test results there are uncertainties for the values of the model parameters. This affects the total PoF with respect to LCF crack initiation. We employ the fully parametric bootstrap sampling procedure in conjunction with the percentile method – confer [5] – to consider these additional uncertainties. We used 2,000 bootstrap samples which were obtained from the maximum likelihood estimation. These samples are different parameter realizations of the probabilistic model for LCF and describe the distributions of the model parameters. Using these parameter realizations in conjunction with the FEA postprocessing results in 2,000 values for the Weibull shape and scale parameter. Then, the law of total probability yields the total PoF with respect to LCF crack initiation. Note that computationally most expensive parts of the FEA postprocessing are identifying the blade’s surface and computing the surface integral in (6) for the bootstrap samples.

Figure 2 shows cumulative Weibull distributions corresponding to 72 of the 2,000 bootstrap samples. Each black curve is one parameter realization of the probabilistic model and yields different values for the PoF depending on multiples N^* of the maximum likelihood value of η . The law of total probability results in the total PoF (red curve). For $N^* = 0.0796$ the total PoF is 3.032%, for example. From a design perspective one decides which PoF is acceptable and then chooses the corresponding number of allowable shutdown and operating cycles. Figure 3 shows the hazard density on the turbine’s surface, where the red regions are areas with higher risk for crack initiation. Blue regions in some cases may indicate overengineered areas of the turbine blade. To obtain a final judgment on overengineered areas a more complete design perspective is needed which includes performance and efficiency criteria, for example.

⁴The results were provided by Siemens AG.

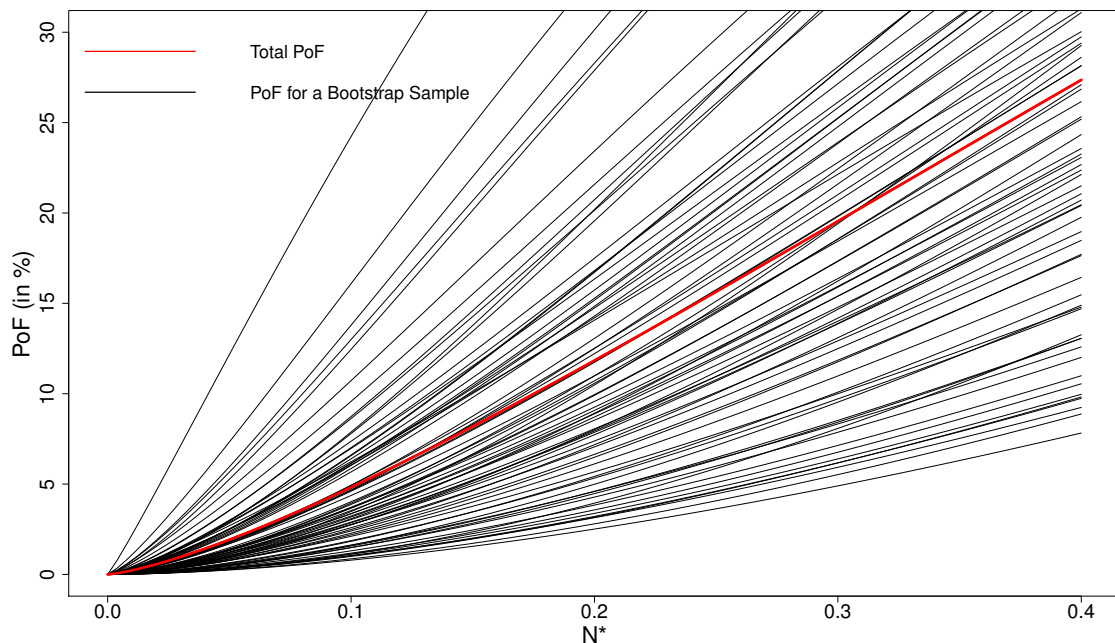


Figure 2: Total PoF (red) due to LCF crack initiation and single PoF curves (cumulative Weibull distributions, black) corresponding to 72 of the 2,000 bootstrap samples. N^* are multiples of the maximum likelihood value for the Weibull scale η .

4 Conclusion

In this work, we computed the hazard density and the total PoF of a turbine blade under cyclic loading due to LCF crack initiation on the surface. The computations are based on the probabilistic model for LCF as proposed in [1], [2] and [3]. In order to consider thermomechanical loading and uncertainties due to LCF test data we extended the model by a temperature model and included the percentile bootstrap method, respectively.

In future, we plan to consider information on local strain gradients. LCF cracks initiating at the surface grow into the component, where a different local strain field may result to a different speed of crack growth, confer [4]. Moreover, we plan to extend the probabilistic model to consider HCF, TMF and non-stationary FEA. Finally, note that the model can be used to optimize the total PoF with respect to the shape Ω , i.e. to find a design Ω under certain constraints such that surface integrals of the form of (6) are minimized. This is also called optimal reliability, confer [2].

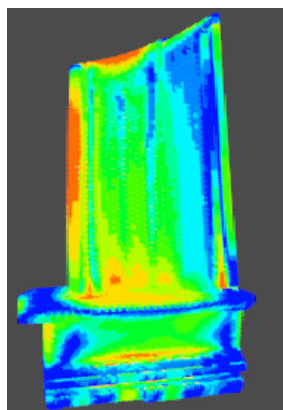


Figure 3: Hazard density of the analyzed turbine blade: Red areas show critical regions and blue ones may indicate overengineered regions of the blade.

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