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Modelling the dynamics of the students academic performance in the German region of North Rhine-Westphalia

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Abstract

Academic underachievement is a concern of paramount importance in Europe, where around 15% of the students in the last courses in high school do not achieve the minimum knowledge academic requirement. In this paper, we propose a model based on a system of differential equations to study the dynamics of the students academic performance in the German region of North Rhine-Westphalia. This approach is supported by the idea that both, good and bad study habits, are a mixture of personal decisions and influence of classmates. This model may permit to forecast trends in the next few years.

Key words: Academic Performance, Modelling, System of Differential Equations, Forecasting in Social Sciences.

1 Introduction

In many countries of the European Union, in the last courses of high school, the rates of academic underachievement are at very worrying levels [1, 2, 3, 4, 5]. The concern about the high level of academic underachievement is completely justified, not only by the high

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rates but also by the negative effects on the country's economic development, especially in the unemployment and its serious consequences. Nowadays, the job opportunities of people depend on their qualification, their ability to acquire, use and interprete the information, including their skills to adapt the new knowledge to a very demanding and competitive society in constant change. In order to acquire them, students go to basic schools first and high schools later, learning the contents determined in the corresponding legislations.

The main goal of the last high school courses is to provide the students a proper educational training to consolidate the intellectual maturity of the pupils, increasing their specific knowledge as well as boosting the development of abilities that help them to join up either the labor market or higher studies. For all these reasons, this educational level is considered a milestone to students because it represents a period to make important decisions about academic and professional future.

According to the Vygotsky learning theories [6, 7] and the recent studies published by Christakis and Fowler [8], habits and behavior may be socially transmitted, in particular, academic and study habits.

Taking into account this approach, in this paper, we are going to focus in the German region of North Rhine-Westphalia and propose a model to study the evolution of the students academic performance in the last three courses of the high school (levels 11, 12 and 13) before accessing to the university by most of the students, using techniques of mathematical epidemiology. This approach may be of relevant interest because a new studies plan will come into force next year in North Rhine-Westphalia and the model forecasting academic results could be compared to the real ones corresponding to the new plan in order to evaluate if the change has been as good as expected.

Some examples of social problems approached using type-epidemiological mathematical models are encountered in obesity [9, 10], alcoholism [11], drug abuse [12], shopaholism [13], spread of ideas [14], evaluation of law effects on societies [15], and so on.

2 Model building

2.1 Available data

We say that a student *promotes* if, in case the course finishes now, he or she will pass to the next level or graduate satisfying the current legislation into force in North Rhine-Westphalia. Otherwise, this student is in *non-promote* group. The legislation establishes that the grades in North Rhine-Westphalia are "very good" (1), "good" (2), "satisfactory" (3), "sufficient" (4), "bad" (5) and "very bad" (6). A student in level 11 and 12 does not promote to next level if he/she has in 2 or more main subjects (like Maths, Physics, German, English) or in 3 or more minor subjects (like music, arts, sports), a grade of 5 or 6. In case the student is in the last level (level 13), he/she has to pass all the subjects to obtain the grade [16, 17].

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The available data that we have considered in this paper correspond to the academic results belonging to the students of the last three courses of high schools during the academic years from $2006-2007$ to $2010-2011$, in both, state and private high schools all over North Rhine-Westphalia, divided by gender, level and promote/non-promote. The corresponding data can be seen in Table 1 [18].

GIRLS		2006–2007	2007–2008	2008–2009	2009–2010	2010–2011
Level 11	% Promote	19.37	19.09	19.1	19.24	18.27
	% Non–Promote	0.81	0.67	0.59	0.53	0.44
Level 12	% Promote	18.23	17.96	18.15	17.77	18.29
	% Non–Promote	0.75	0.68	0.58	0.47	0.47
Level 13	% Promote	15.34	15.96	15.94	16.25	16.44
	% Non–Promote	0.25	0.25	0.19	0.19	0.17
BOYS		**2006–2007**	**2007–2008**	**2008–2009**	**2009–2010**	**2010–2011**
Level 11	% Promote	16.05	15.92	15.95	16.3	15.87
	% Non–Promote	0.96	0.88	0.81	0.73	0.6
Level 12	% Promote	14.7	14.73	14.77	14.72	15.21
	% Non–Promote	0.85	0.81	0.67	0.67	0.64
Level 13	% Promote	12.38	12.77	13.04	12.94	13.39
	% Non–Promote	0.31	0.28	0.21	0.19	0.21

Table 1: The available data corresponding to levels 11, 12 and 13, in both, state and private high schools all over North Rhine-Westphalia from academic year $2006-2007$ to $2010-2011$ divided by gender, level and promote/non-promote over the total number of students in the three levels.

2.2 The type-epidemiological model

We build our mathematical model following an epidemiological approach considering that the academic performance of a student, Girl (G) or Boy (B), is a mixture of her/his own study habits and his/her classmates study habits, good or bad. In our model, we assume that the transmission of good and bad academic habits is caused by the social contact between students who belong to the same academic level [8, 7, 19].

The subpopulations of the model will be (time t in years and $i = 1$ for level 11, $i = 2$ for level 12 and $i = 3$ for level 13):

- $G_i = G_i(t)$ is the number of girls of level i who promote at time instant t.
- $B_i = B_i(t)$ is the number of boys of level i who promote at time instant t.
- $\overline{G}_i = \overline{G}_i(t)$ is the number of girls of level i who do not promote at time instant t.
- $\overline{B}_i = \overline{B}_i(t)$ is the number of boys of level i who do not promote at time instant t.

Furthermore, we consider the following assumptions to design the model:

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- Let us assume a homogeneous population mixing, i.e., each student can contact with any other student in his/her class [20].

- *Negative autonomous decision*: For each academic level, $i = 1, 2, 3$, students belonging to the promotable groups G_i or B_i may change their personal study habits and this change may lead them to obtain bad academic results, moving to $\overline{G}_i$ or $\overline{B}_i$. We assume that this transition is proportional to the number of pupils in G_i and B_i, and it is modelled by the linear terms $\alpha_i^G G_i$ and $\alpha_i^B B_i$. According to educational experts, it is assumed that the academic attitude is different in the same educational level depending on gender: girls are usually more responsible for their academic performance than boys [21]. This leads us to suppose the following restrictions:

$$\alpha_1^G < \alpha_1^B,\ \alpha_2^G < \alpha_2^B,\ \alpha_3^G < \alpha_3^B. \tag{1}$$

  In addition we will assume that:

$$\alpha_1^G > \alpha_2^G > \alpha_3^G,\ \alpha_1^B > \alpha_2^B > \alpha_3^B, \tag{2}$$

  because students in the higher levels are more mature than their mates in the lower levels [21].

- *Negative habits transmission*: For each academic level, $i = 1, 2, 3$, students in G_i or B_i may move to the non–promotable group, $\overline{G}_i$ or $\overline{B}_i$ respectively, due to the negative influence transmitted by encounters between students (girls and boys) in the non–promotable group in the same academic level. Hence, these transitions are modelled by the nonlinear terms $\beta_i^{G\overline{G}} G_i \overline{G}_i + \beta_i^{G\overline{B}} G_i \overline{B}_i$ and $\beta_i^{B\overline{G}} B_i \overline{G}_i + \beta_i^{B\overline{B}} B_i \overline{B}_i$, where $\beta_i^{G\overline{G}}$, $\beta_i^{G\overline{B}}$, $\beta_i^{B\overline{G}}$ and $\beta_i^{B\overline{B}}$ are the corresponding transmission rates where the first letter in the superindexes denotes the group susceptible to acquire bad study habits and the second one denotes the group that transmit those bad study habits. All specific factors and social encounters involved in the transmission of the bad academic habits are embedded in β parameters.

- *Positive autonomous decision*: Analogously to *negative autonomous decision*, students belonging to the non–promotable groups may change their personal behavior towards their study habits and this change may lead the students to improve their academic results, moving to G_i or B_i. We assume that this transition is proportional to the number of pupils in $\overline{G}_i$ and $\overline{B}_i$, and it is modelled by the linear terms $\gamma_i^G \overline{G}_i$ and $\gamma_i^B \overline{B}_i$.

- *Positive habits transmission*: Students in non–promotable group may move to the promotable groups due to the positive influence transmitted in the encounters between students (girls and boys) in the promotable group in the same academic level.

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Hence, these transitions are modelled by the nonlinear terms $\delta_i^{\overline{G}G}\overline{G}_iG_i+\delta_i^{\overline{G}B}\overline{G}_iB_i$ and $\delta_i^{\overline{B}G}\overline{B}_iG_i+\delta_i^{\overline{B}B}\overline{B}_iB_i$. The interpretation of the transmission rate parameters is the same as in the *negative habits transmission*.

- *Passing courses and graduation*: The students in G_i and B_i, in September, transit automatically to next level G_{i+1} and B_{i+1}, respectively, for $i = 1, 2$. Students in G_3 and B_3 will graduate in September. These transitions are modelled by εG_1, εG_2, εG_3, εB_1, εB_2, εB_3, where

$$\varepsilon = \begin{cases} 1 & \text{if} \quad \dfrac{9}{12}+j \le t \le \dfrac{10}{12}+j, \\ 0 & \text{otherwise,} \end{cases}$$

  where $j = 0, 1, 2, 3, 4$, correspond to academic years 2006–2007, …, 2010–2011, respectively.

- *Abandon*: For each academic level, $i = 1, 2, 3$, a proportion of the students in $\overline{G}_i$ or $\overline{B}_i$ with bad academic results may leave their studies by autonomous decision. This situation is modelled by the linear terms $\eta_i^G\overline{G}_i$ and $\eta_i^B\overline{B}_i$. We also assume that these transitions are proportional to the number of pupils in $\overline{G}_i$ and $\overline{B}_i$.

- *Access*: New students enter into the level 11 in the month of September in the promotable groups of girls and boys. It is modelled by the functions

$$\sigma^G = \begin{cases} \tau^G & \text{if} \quad \dfrac{9}{12}+j \le t \le \dfrac{10}{12}+j, \\ 0 & \text{otherwise,} \end{cases} \quad \sigma^B = \begin{cases} \tau^B & \text{if} \quad \dfrac{9}{12}+j \le t \le \dfrac{10}{12}+j, \\ 0 & \text{otherwise,} \end{cases}$$

  where $j = 0, 1, 2, 3, 4$, correspond to academic years 2006–2007, …, 2010–2011, respectively, and τ^G and τ^B to be determined.

Thus, under the above assumptions we build the nonlinear system of ordinary differential equations (3)-(5) in order to describe the dynamics of students academic performance in the German region of North Rhine-Westphalia.

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$$
\begin{aligned}
G_1'(t) &= \sigma^G - \varepsilon G_1(t) - \alpha_1^G G_1(t) + \gamma_1^G \overline{G}_1(t) \\
&- \left[\beta_1^{G\overline{G}} G_1(t)\tfrac{\overline{G}_1(t)}{T(t)} + \beta_1^{G\overline{B}} G_1(t)\tfrac{\overline{B}_1(t)}{T(t)}\right] + \left[\delta_1^{\overline{G}G}\overline{G}_1(t)\tfrac{G_1(t)}{T(t)} + \delta_1^{\overline{G}B}\overline{G}_1(t)\tfrac{B_1(t)}{T(t)}\right], \\
\overline{G}_1'(t) &= \alpha_1^G G_1(t) - \gamma_1^G \overline{G}_1(t) - \eta_1^G \overline{G}_1(t) \\
&+ \left[\beta_1^{G\overline{G}} G_1(t)\tfrac{\overline{G}_1(t)}{T(t)} + \beta_1^{G\overline{B}} G_1(t)\tfrac{\overline{B}_1(t)}{T(t)}\right] - \left[\delta_1^{\overline{G}G}\overline{G}_1(t)\tfrac{G_1(t)}{T(t)} + \delta_1^{\overline{G}B}\overline{G}_1(t)\tfrac{B_1(t)}{T(t)}\right], \\
G_2'(t) &= \varepsilon G_1(t) - \varepsilon G_2(t) - \alpha_2^G G_2(t) + \gamma_2^G \overline{G}_2(t) \\
&- \left[\beta_2^{G\overline{G}} G_2(t)\tfrac{\overline{G}_2(t)}{T(t)} + \beta_2^{G\overline{B}} G_2(t)\tfrac{\overline{B}_2(t)}{T(t)}\right] + \left[\delta_2^{\overline{G}G}\overline{G}_2(t)\tfrac{G_2(t)}{T(t)} + \delta_2^{\overline{G}B}\overline{G}_2(t)\tfrac{B_2(t)}{T(t)}\right], \\
\overline{G}_2'(t) &= \alpha_2^G G_2(t) - \gamma_2^G \overline{G}_2(t) - \eta_2^G \overline{G}_2(t) \\
&+ \left[\beta_2^{G\overline{G}} G_2(t)\tfrac{\overline{G}_2(t)}{T(t)} + \beta_2^{G\overline{B}} G_2(t)\tfrac{\overline{B}_2(t)}{T(t)}\right] - \left[\delta_2^{\overline{G}G}\overline{G}_2(t)\tfrac{G_2(t)}{T(t)} + \delta_2^{\overline{G}B}\overline{G}_2(t)\tfrac{B_2(t)}{T(t)}\right], \\
G_3'(t) &= \varepsilon G_2(t) - \varepsilon G_3(t) - \alpha_3^G G_3(t) + \gamma_3^G \overline{G}_3(t) \\
&- \left[\beta_3^{G\overline{G}} G_3(t)\tfrac{\overline{G}_3(t)}{T(t)} + \beta_3^{G\overline{B}} G_3(t)\tfrac{\overline{B}_3(t)}{T(t)}\right] + \left[\delta_3^{\overline{G}G}\overline{G}_3(t)\tfrac{G_3(t)}{T(t)} + \delta_3^{\overline{G}B}\overline{G}_3(t)\tfrac{B_3(t)}{T(t)}\right], \\
\overline{G}_3'(t) &= \alpha_3^G G_3(t) - \gamma_3^G \overline{G}_3(t) - \eta_3^G \overline{G}_3(t) \\
&+ \left[\beta_3^{G\overline{G}} G_3(t)\tfrac{\overline{G}_3(t)}{T(t)} + \beta_3^{G\overline{B}} G_3(t)\tfrac{\overline{B}_3(t)}{T(t)}\right] - \left[\delta_3^{\overline{G}G}\overline{G}_3(t)\tfrac{G_3(t)}{T(t)} + \delta_3^{\overline{G}B}\overline{G}_3(t)\tfrac{B_3(t)}{T(t)}\right],
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
B_1'(t) &= \sigma^B - \varepsilon B_1(t) - \alpha_1^B B_1(t) + \gamma_1^B \overline{B}_1(t) \\
&- \left[\beta_1^{B\overline{G}} B_1(t)\tfrac{\overline{G}_1(t)}{T(t)} + \beta_1^{B\overline{B}} B_1(t)\tfrac{\overline{B}_1(t)}{T(t)}\right] + \left[\delta_1^{\overline{B}G}\overline{B}_1(t)\tfrac{G_1(t)}{T(t)} + \delta_1^{\overline{B}B}\overline{B}_1(t)\tfrac{B_1(t)}{T(t)}\right], \\
\overline{B}_1'(t) &= \alpha_1^B B_1(t) - \gamma_1^B \overline{B}_1(t) - \eta_1^B \overline{B}_1(t) \\
&+ \left[\beta_1^{B\overline{G}} B_1(t)\tfrac{\overline{G}_1(t)}{T(t)} + \beta_1^{B\overline{B}} B_1(t)\tfrac{\overline{B}_1(t)}{T(t)}\right] - \left[\delta_1^{\overline{B}G}\overline{B}_1(t)\tfrac{G_1(t)}{T(t)} + \delta_1^{\overline{B}B}\overline{B}_1(t)\tfrac{B_1(t)}{T(t)}\right], \\
B_2'(t) &= \varepsilon B_1(t) - \varepsilon B_2(t) - \alpha_2^B B_2(t) + \gamma_2^B \overline{B}_2(t) \\
&- \left[\beta_2^{B\overline{G}} B_2(t)\tfrac{\overline{G}_2(t)}{T(t)} + \beta_2^{B\overline{B}} B_2(t)\tfrac{\overline{B}_2(t)}{T(t)}\right] + \left[\delta_2^{\overline{B}G}\overline{B}_2(t)\tfrac{G_2(t)}{T(t)} + \delta_2^{\overline{B}B}\overline{B}_2(t)\tfrac{B_2(t)}{T(t)}\right], \\
\overline{B}_2'(t) &= \alpha_2^B B_2(t) - \gamma_2^B \overline{B}_2(t) - \eta_2^B \overline{B}_2(t) \\
&+ \left[\beta_2^{B\overline{G}} B_2(t)\tfrac{\overline{G}_2(t)}{T(t)} + \beta_2^{B\overline{B}} B_2(t)\tfrac{\overline{B}_2(t)}{T(t)}\right] - \left[\delta_2^{\overline{B}G}\overline{B}_2(t)\tfrac{G_2(t)}{T(t)} + \delta_2^{\overline{B}B}\overline{B}_2(t)\tfrac{B_2(t)}{T(t)}\right], \\
B_3'(t) &= \varepsilon B_2(t) - \varepsilon B_3(t) - \alpha_3^B B_3(t) + \gamma_3^B \overline{B}_3(t) \\
&- \left[\beta_3^{B\overline{G}} B_3(t)\tfrac{\overline{G}_3(t)}{T(t)} + \beta_3^{B\overline{B}} B_3(t)\tfrac{\overline{B}_3(t)}{T(t)}\right] + \left[\delta_3^{\overline{B}G}\overline{B}_3(t)\tfrac{G_3(t)}{T(t)} + \delta_3^{\overline{B}B}\overline{B}_3(t)\tfrac{B_3(t)}{T(t)}\right], \\
\overline{B}_3'(t) &= \alpha_3^B B_3(t) - \gamma_3^B \overline{B}_3(t) - \eta_3^B \overline{B}_3(t) \\
&+ \left[\beta_3^{B\overline{G}} B_3(t)\tfrac{\overline{G}_3(t)}{T(t)} + \beta_3^{B\overline{B}} B_3(t)\tfrac{\overline{B}_3(t)}{T(t)}\right] - \left[\delta_3^{\overline{B}G}\overline{B}_3(t)\tfrac{G_3(t)}{T(t)} + \delta_3^{\overline{B}B}\overline{B}_3(t)\tfrac{B_3(t)}{T(t)}\right],
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
T(t) &= G_1(t) + \overline{G}_1(t) + B_1(t) + \overline{B}_1(t) + G_2(t) + \overline{G}_2(t) + B_2(t) + \overline{B}_2(t) \\
&+ G_3(t) + \overline{G}_3(t) + B_3(t) + \overline{B}_3(t).
\end{aligned}
\tag{5}
$$

The flow diagram, associated to the above model, is plotted in Figure 1.

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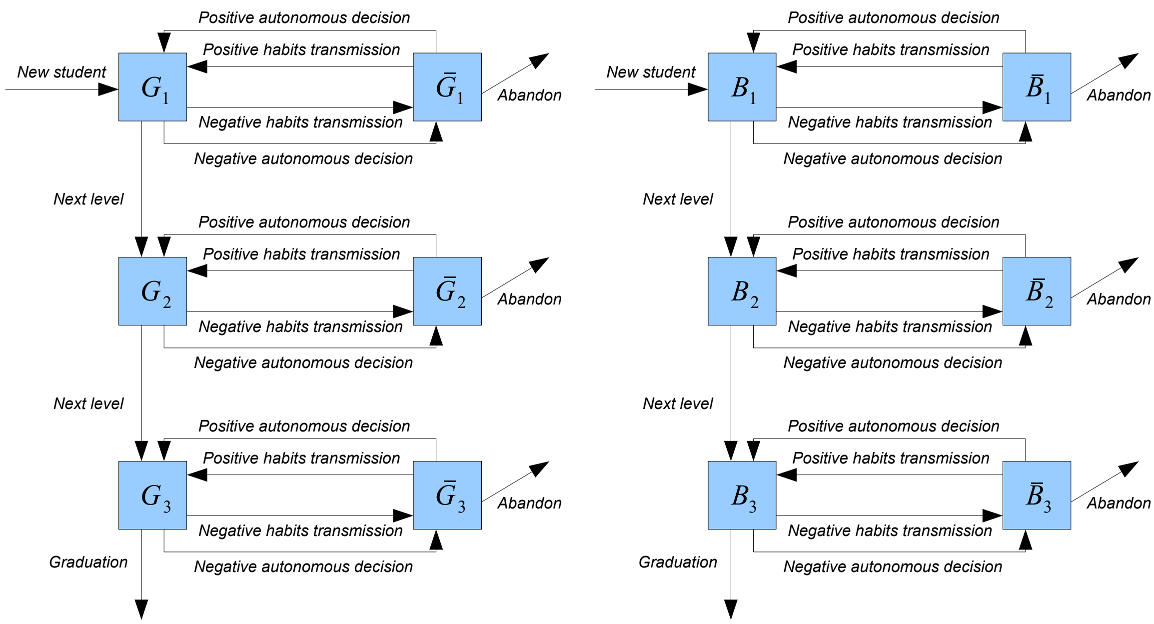

Figure 1: Flow diagram of the model (3)-(5). The boxes represent the students depending on their gender, level and academic results. The arrows denote the transit of students labelled by the cause of the flow.

3 Scaling, fitting and predictions

Data in Table 1 are in percentages meanwhile model (3)-(5) is referred to number of students. It leads us to transform (scaling) the model into the same units as data in order to fit the model with the data. To do that, we follow the techniques developed in [22, 23] about how to scale models where the population is varying in size. Here, we are not going to show the process and the scaled model because it is a technical transformation, the resulting equations are more complex and longer and does not provide extra information about the model. Moreover, the scaled model has the same parameters as the non-scaled model with the same meaning. In order to avoid introducing new notation, we are going to consider that the subpopulations $G_1(t)$, $\overline{G}_1(t)$, $B_1(t)$, $\overline{B}_1(t)$, $G_2(t)$, $\overline{G}_2(t)$, $B_2(t)$, $\overline{B}_2(t)$, $G_3(t)$, $\overline{G}_3(t)$, $B_3(t)$, $\overline{B}_3(t)$ correspond to the percentage of Girls and Boys in the promotable and non–promotable groups in the levels 11, 12 and 13.

Now, compute the model parameters that best fit the scaled model with the available data collected in Table 1 in the mean square sense. Computations have been carried out with *Mathematica 8.0* [24] and the estimated model parameters are:

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- *Negative autonomous decision*:
  - Girls per level: $\alpha_1^G = 0.00257431$, $\alpha_2^G = 0.000479681$, $\alpha_3^G = 0.0000980351$.
  - Boys per level: $\alpha_1^B = 0.000518445$, $\alpha_2^B = 0.000462886$, $\alpha_3^B = 0.0000783883$.
- *Negative habits transmission*:
  - Girls per level: $\beta_1^{G\overline{G}} = 0.128823$, $\beta_1^{G\overline{B}} = 0.146999$, $\beta_2^{G\overline{G}} = 0.115597$, $\beta_2^{G\overline{B}} = 0.0940018$, $\beta_3^{G\overline{G}} = 0.128018$, $\beta_3^{G\overline{B}} = 0.0465132$.
  - Boys per level: $\beta_1^{B\overline{G}} = 0.124969$, $\beta_1^{B\overline{B}} = 0.0247756$, $\beta_2^{B\overline{G}} = 0.0406373$, $\beta_2^{B\overline{B}} = 0.0893315$, $\beta_3^{B\overline{G}} = 0.115285$, $\beta_3^{B\overline{B}} = 0.0713746$.
- *Positive autonomous decision*:
  - Girls per level: $\gamma_1^G = 0.0598649$, $\gamma_2^G = 0.138232$, $\gamma_3^G = 0.00441141$.
  - Boys per level: $\gamma_1^B = 0.0254583$, $\gamma_2^B = 0.0407112$, $\gamma_3^B = 0.143022$.
- *Positive habits transmission*:
  - Girls per level: $\delta_1^{G\overline{G}} = 0.0628747$, $\delta_1^{G\overline{B}} = 0.117906$, $\delta_2^{G\overline{G}} = 0.0162307$, $\delta_2^{G\overline{B}} = 0.0217844$, $\delta_3^{G\overline{G}} = 0.064252$, $\delta_3^{G\overline{B}} = 0.0722602$.
  - Boys per level: $\delta_1^{B\overline{G}} = 0.0831484$, $\delta_1^{B\overline{B}} = 0.0396256$, $\delta_2^{B\overline{G}} = 0.14784$, $\delta_2^{B\overline{B}} = 0.0560535$, $\delta_3^{B\overline{G}} = 0.0199681$, $\delta_3^{B\overline{B}} = 0.0505348$.
- *Abandon*:
  - Girls per level: $\eta_1^G = 0.0899652$, $\eta_2^G = 0.0620594$, $\eta_3^G = 0.118145$.
  - Boys per level: $\eta_1^B = 0.111194$, $\eta_2^B = 0.0445628$, $\eta_3^B = 0.0235689$.
- *Access*:
  - Girls: $\tau^G = 0.121096$.
  - Boys: $\tau^B = 0.12517$.

Once the parameters are estimated, we are able to give predictions of each group and level over the next few years by computing the solutions of the model for values of time t in the forthcoming future. The results can be seen in Figure 2.

In Table 2 we present the prediction of percentage of non–promote students for the next four courses.

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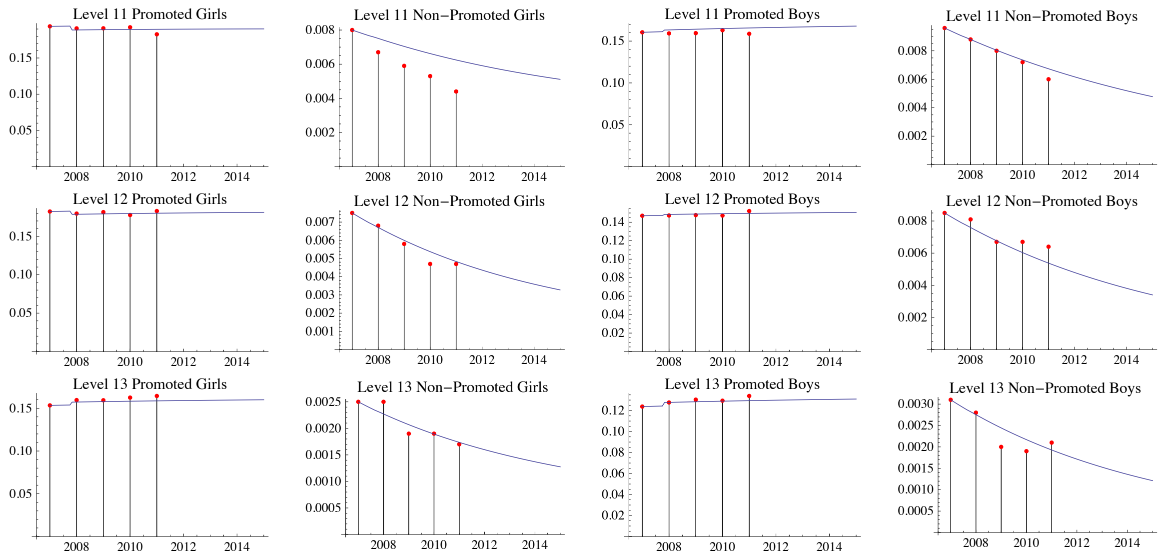

Figure 2: Graph representing the model fitting and the predictions until the course 2014-2015. Note that there is a decreasing trend in the non-promotable groups.

4 Conclusion

In this paper we present a model to study the dynamics of the students academic performance in the German region of North Rhine-Westphalia. In this model we divide the students by gender and academic levels, and it is based on the assumption that both, good and bad study habits, are a mixture of personal decisions and influence of classmates. Using data of the students academic performance, we estimate the model parameters fitting the model with the data. Thus, we can predict the students academic performance in the next few years. In Figure 2, it is expected that the decreasing trend in all non–promotable groups continues in the next years. For instance, in the course 2014-2015 less than 2% of the students will not promote (see Table 2).

This model will allow us to compare the performance of the coming new academic plan to this one in order to evaluate if the change is as good as expected.

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	2011–2012	2012–2013	2013–2014	2014–2015
Level 11, Non–Promote girls	0.591 %	0.561 %	0.535 %	0.511 %
Level 12, Non–Promote girls	0.436 %	0.395 %	0.359 %	0.327 %
Level 13, Non–Promote girls	0.16 %	0.148 %	0.137 %	0.127 %
Level 11, Non–Promote boys	0.617 %	0.566 %	0.52 %	0.477 %
Level 12, Non–Promote boys	0.479 %	0.427 %	0.381 %	0.34 %
Level 13, Non–Promote boys	0.171 %	0.152 %	0.136 %	0.121 %
TOTAL	2.455 %	2.25 %	2.067 %	1.905 %

Table 2: Prediction for the next four courses of the percentage of non-promoted students per gender and level, and the total. Note that there is a decreasing trend over the time in all levels with independence of gender. Also, the percentages decrease when the level increases. There are minor differences between boys and girls figures.

2070.

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